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MULTIVARIATE SPATIAL MODELLING AND PREDICTION OF METEOROLOGICAL DATA

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ABSTRACT

This study proposed a multivariate spatial modelling and prediction under which one specifies a mixed effects model and the joint covariance models. Joint predictions through Cokriging methods and multiple regression models were achieved. Both approaches were illustrated using data on wind speed and wind direction obtained from World Meteorological Organisation stations (WMO), from meteorological services of Nigeria annual summary of observations. Comparing the results from the two methods reveals that cokriging method has lower absolute bias for all the distances and directions under consideration for both the wind speed and wind direction except for wind speed in distance 500 km southeast of the initial location.

Keywords: Cokriging, median polish, coarse mapping, semivariogram, scatter plot matrix.

INTRODUCTION

Predictions of wind speed and wind direction are important issues in environmental monitoring. The relationship between wind speed and wind direction has not been established. Both theory and data suggest strongly that for every wind speed there is a corresponding wind direction, and that wind speed of zero Beaufort force is said to be directionless (WMO, 1961)

We present a new approach to spatial joint modelling and prediction of meteorological data using multivariate cokriging methods. In this study, Nigeria was considered as a region. The data set is an extract from annual summary of observations published by meteorological services of Nigeria. The raw data is counts of wind speeds and wind directions. Wind speeds in the data are reported as Beaufort force of 5-4, 3-1, and 0, which are denoted by y_1, y_2, and y_3 respectively. The corresponding wind direction to any Beaufort force is N, NE, E, SE, S, SW, W, and NW. However, in this study N, NE, E are used and denoted by x_1, x_2, and x_3 respectively. The locations used are those in which data is available for both wind speed and wind direction.

The objectives of the study are to formulate a mixed effects model for the processes, and to produce statistically sound and physically motivated predictions of both wind speeds and wind directions. To illustrate the approach, detrended data for a month was used to provide predictions after grouping the sites into bins of equal intervals of 100km.

2 LITERATURE REVIEW

To enhance understanding and predictability of meteorological conditions, statistical models have been proposed. Notable among them are: Brown et al (1994) on multivariate spatial interpolation and exposure to air pollutants, and Royle and Berliner (1999) on hierarchical approach to multivariate spatial modelling and prediction. Statistical modelling of Ozone using cokriging methods was considered by Myers (1982). Royle et al (2002) on spatial modelling framework for wetland data proposed a two-state model. That is, basins are either wet or dry so that the state is observed. The estimation and predictions were achieved through Markov chain Monte Carlo. Gotway and Hartford (1996) on geostatistical methods for incorporating auxiliary information in the prediction of spatial variables used cokriging methods and universal kriging models. They obtained the mean squared errors from cross- validation of the three approaches, with cokriging giving the least mean squared error.

2.1 Multivariate Analysis:

Multivariate analysis deals with observations on more than one variable where there is some inherent interdependence between the variables. In this article, three sets of counts were presented for wind speed and wind direction made at each of the fixed twenty six monitoring sites of World Meteorological Organization (WMO) in Nigeria. The response at each time is then a 6x26 matrix of counts. Figure 1 shows a coarse mapping of twenty six data sites.

2.2 Multivariate Models

The model is of the form:

$$\boldsymbol{Y}(s_i) = \boldsymbol{X}(s_i) + \epsilon(s_i) \quad (2.1)$$

Where Y and X are jointly observed vectors and s_i's are the sites and ε is random error component.

2.3 Cokriging

Consider fields for which two variables have been jointly measured with number of counts, say, N_1 and N_2 measuring $y(s_i)$ and $x(s_i)$ respectively, where counts N_1 may be less than N_2. With the above assumptions in mind, the mean of the processes for all locations will be:

$$E[y(s_i)] = \mu_y(s_i); \quad i\, 1, 2, ---, N \tag{2.2}$$

$$E[x(s_i)] = \mu_x(s_i) \tag{2.3}$$

The covariance matrix is of the form,

$$\begin{pmatrix} L_{yy}(s_i) & L_{yx}(s_i) \\ L_{xy}(s_i) & L_{xx}(s_i) \end{pmatrix} \tag{2.4}$$

where $L_{yy}(s_i)$ and $L_{xx}(s_i)$ are the marginal covariance matrices sometimes called auto-covariance matrices and $L_{yx}(s_i)$ and $L_{xy}(s_i)$ are the cross – covariance matrices depicting spatial relationships between $y(s_i)$ and $x(s_i)$ respectively, and it is not necessary that

$$L_{yx}(s_i) = L_{xy}(s_i)$$

Since the process is jointly observed with mean and covariance, we can represent the observed process as;

$$E\begin{pmatrix} y(s_i) \\ x(s_i) \end{pmatrix} = \begin{pmatrix} \mu_y(s_i) \\ \mu_x(s_i) \end{pmatrix}; \qquad \text{and}$$

$$\mathrm{Var}\begin{pmatrix} y(s_i) \\ x(s_i) \end{pmatrix} = \begin{pmatrix} L_{yy}(s_i) & L_{yx}(s_i) \\ L_{xy}(s_i) & L_{xx}(s_i) \end{pmatrix}$$

That is;

$$\begin{pmatrix} y(s_i) \\ x(s_i) \end{pmatrix} \sim \left[\begin{pmatrix} \mu_y(s_i) \\ \mu_x(s_i) \end{pmatrix}, \begin{pmatrix} L_{yy}(s_i) & L_{yx}(s_i) \\ L_{xy}(s_i) & L_{xx}(s_i) \end{pmatrix}\right] \tag{2.5}$$

Adopting the definition of semivariograms and cross-semivariograms for $y(s_i)$ and $x(s_i)$ by Journel and Huijbreghts (1978), the semivariograms and cross-semivariograms for $y(s_i)$ and $x(s_i)$ respectively are given by:

$$\hat{V}_{yy}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left(y(s_{i+h}) - y(s_i)\right)^2 \tag{2.6}$$

$$\hat{V}_{xx}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left(x(s_{i+h}) - x(s_i)\right)^2 \tag{2.7}$$

$$\hat{V}_{yx}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left(y(s_{i+h}) - y(s_i)\right)\left(x(s_{i+h}) - x(s_i)\right) \tag{2.8}$$

$$\hat{V}_{xy}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left(x(s_{i+h}) - x(s_i)\right)\left(y(s_{i+h}) - y(s_i)\right) \tag{2.9}$$

where $N(h)$ is the number of pairs distance h apart.

3.1 Cokriging Modelling:

We take the view that the processes are continuous, and make observations at distinct locations (s_i). We assume that the actual counts of $y(s_i)$ and $x(s_i)$ constitute a realization of a random process at all the monitoring locations. The model gives the variation in the observations:

$$y_r(s_i) = \mu(s_i) + x_m(s_i) + \mathrm{e}(s_i) \tag{3.1}$$

where $\mu(s_i)$ represents the mean field locations (s_i), $y(s_i)$ and $x(s_i)$ are spatial processes observed jointly at all the monitoring locations, and $\mathrm{e}(s_i)$ represents measurement error and small scale spatial variability.

3.2 Predictions

Assume that $y(s_i)$ and $x(s_i)$ are jointly distributed as in equation (2.5). Suppose that we want to predict $(y(s_0), x(s_0))$ after observing $(y(s_i), x(s_i))$ where $i = 1, 2, ---, N$ at all the monitoring locations.

The predictor of $Y(s_0)$ is of the form:

$$\hat{Y}(s_0) = \sum_{r=1}^{N1} a_r y_r(s_i) + \sum_{m=1}^{N2} a_m x_m(s_i) \tag{3.2}$$

where N_1 and N_2 are the levels of $y(s_i)$ and $x(s_i)$ respectively used in the estimation of cokriged values. Similarly,

The predictor of $\mathbf{X}(s_0)$ is of the form:

$$\hat{X}(s_0) = \sum_{r=1}^{N1} b_r y_r(s_i) + \sum_{m=1}^{N2} b_m x_m(s_i) \tag{3.3}$$

The joint predictor $(y(s_0), x(s_0))$ is of the form:

$$\left(\hat{Y}(s_0), \hat{X}(s_0)\right) = \left(\sum_{r=1}^{N1} a_r y_r(s_i) + \sum_{m=1}^{N2} a_m x_m(s_i), \sum_{r=1}^{N1} b_r y_r(s_i) + \sum_{m=1}^{N2} b_m x_m(s_i)\right) \tag{3.5}$$

where a_r, a_m, b_r, and b_m are chosen to minimize the mean squared prediction error subject to the unbiasedness condition,

$$E\left[\left(\hat{y}(s_0), \hat{x}(s_0)\right) - \left(y(s_0), x(s_0)\right)\right] = 0 \tag{3.5}$$

The resulting predictor has minimum variance in the class of linear unbiased predictors and is often referred to as best linear unbiased predictor (BLUP) (Gotway and Hartfield, 1996). Using the method of Lagrange multiplier techniques to minimize the mean squared prediction errors subject to the unbiasedness conditions in equation (3.13) gives the joint cokriging system:

$$\left.\begin{aligned}
&\sum_{r=1}^{N_1} a_r L_{yy}(s_i) + \sum_{m=1}^{N_2} a_m L_{yx}(s_i) + \sum_{r=1}^{N_1} b_r L_{yy}(s_i) + \sum_{m=1}^{N_2} b_m L_{yx}(s_i) - m_1 = L_{yy}(s_i) + L_{yx}(s_i) \\
&\sum_{r=1}^{N_1} a_r L_{xy}(s_i) + \sum_{m=1}^{N_2} a_m L_{xx}(s_i) + \sum_{r=1}^{N_1} b_r L_{xy}(s_i) + \sum_{m=1}^{N_2} b_m L_{xx}(s_i) - m_2 = L_{xx}(s_i) + L_{xy}(s_i) \\
&\sum_{r=1}^{N_1} a_r L_{yy}(s_i) + \sum_{m=1}^{N_2} a_m L_{yx}(s_i) + \sum_{r=1}^{N_1} b_r L_{yy}(s_i) + \sum_{m=1}^{N_2} b_m L_{yx}(s_i) - m_3 = L_{yy}(s_i) + L_{yx}(s_i) \\
&\sum_{r=1}^{N_1} a_r L_{xy}(s_i) + \sum_{m=1}^{N_2} a_m L_{xx}(s_i) + \sum_{r=1}^{N_1} b_r L_{xy}(s_i) + \sum_{m=1}^{N_2} b_m L_{xx}(s_i) - m_4 = L_{xy}(s_i) + L_{xx}(s_i)
\end{aligned}\right\} \qquad (3.6)$$

with the constraints

$$\left.\begin{aligned}
&\sum_{m=1}^{N_2} a_m = 0 = \sum_{r=1}^{N_1} b_r \\
&\sum_{r=1}^{N_1} a_r = 1 = \sum_{m=1}^{N_2} b_m
\end{aligned}\right\} \qquad (3.7)$$

Details of the minimization procedure can be found in Isah (2008).

The Cokriging variance $\delta_E^2(s_0)$ are:

$$\left.\begin{aligned}
&\sum_{r=1}^{N_1} a_r L_{yy}(s_i) + \sum_{m=1}^{N_2} a_m L_{yx}(s_i) + \sum_{r=1}^{N_1} b_r L_{yy}(s_i) + \sum_{m=1}^{N_2} b_m L_{yx}(s_i) - m_1 = L_{yy}(s_i) + L_{yx}(s_i) \\
&\sum_{r=1}^{N_1} a_r L_{xy}(s_i) + \sum_{m=1}^{N_2} a_m L_{xx}(s_i) + \sum_{r=1}^{N_1} b_r L_{xy}(s_i) + \sum_{m=1}^{N_2} b_m L_{xx}(s_i) - m_2 = L_{xx}(s_i) + L_{xy}(s_i)
\end{aligned}\right\} \qquad (3.8)$$

L_{yy}, L_{yx}, L_{xy} and L_{xx} are covariance matrices for the various models respectively. The coefficients a_r, $\mathrm{a_m}$, $\mathrm{b_r}$, and $\mathrm{b_m}$ and the Lagrange multipliers m_1 and $\mathrm{m_2}$ can be determined using equation (3.8).

3.3 Estimation Error

Cokriging as any other method of estimation involve error. This is due to the fact that the variable to be estimated is somewhat different from the estimated value (Journel and Huijbregts, 1978). The estimation error $e(y_k, x_k)$, is thus defined as the difference between the joint measured and the joint estimated values for the same location (s_i). The computational form is given below:

$$e(y_i, x_i) = [(y_i, x_i)^* - (y_i, x_i)] \qquad (3.9)$$

4.1 Descriptive Statistical Analysis

The data sets used as examples were purposely chosen to illustrate the theoretical concepts presented under the last section of this article. The raw data as described in section 1 were detrended using median polishing technique to enable us use geostatistical technique for analysis. The descriptive statistics of the raw and polished data are given in table 1.

Table 1 shows variation in mean and variance between the two sets of data. A reduction in mean and variance of polished data over the raw data is an indication of removal of trends in the raw data. This makes the raw data ready for geostatistical analysis.

4.2 Joint Cokriging Predictors

The semivariograms and cross- semivariograms for detrended data were computed for various distance using equations (2.6) through (2.9). The results were further used in equations (3.4) and (3.8) respectively to obtain the mean and variance for different distances and directions as presented in table 2 and table 3 respectively.

4.3 Results

We present results from our analysis under the mixed spatial model. The Marginal covariance and cross-covariance matrices depicting spatial relationships were computed using the residuals data in equation (3.4) for the calculation of joint predictors given in table 2, as well as estimated prediction variance matrices in table 3. Results from multivariate regression models were also provided on raw data in table 4. The scatter plot matrix for the joint prediction using equation (3.4) is shown in figure 2.

4.4 Discussion

Table 2 shows that the absolute bias is relatively smaller in distance 100km for wind speed and in distance 500km east for wind direction. Using distance 500km, the bias is relatively larger to the north for both wind speed and wind direction as compared to the other two directions. This implies that there are different covariance structures in different directions. The prediction is better for distance 100km in wind speed, and in distance 500km to the east. The absolute bias from multivariate regression is lower for wind direction. Comparing the results from the two methods reveals that Cokriging method has lower absolute bias for all the distances and directions under consideration for both the wind speed and wind direction except for wind speed in distance 500km southeast.

The estimates from the multivariate regression are inconsistent with that of the Cokriging model since additional information is used in Cokriging (i.e. distance and direction).

In table 3, the first part which is a 6x3 matrix is the matrix of Cokriging variance. The first 3x3 matrix represents the y_i's while the second 3x3 matrix represents the x_i's. The second part of the set is the Lagrange multiplier, each multiplier is for a process being predicted.

5 CONCLUSION

This article has presented two different methods of prediction. The first is geostatistical, and to enable us make comparison a traditional method was also used. Each approach emphasized different aspect as reflected in the models. The distinction between the two approaches is subtle, but important. With the Cokriging methods, more of the secondary information is used by directly incorporating the values of the secondary variable and measuring the degree of spatial association with the primary variable through the cross-variogram. The multivariate analysis provides a means of investigating underlying structure in complex data. To explore the stability of multivariate data analysis, we employed the mvreg. command in stata software for the application of the method.

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Table 1 Descriptive statistics

Site	Mean											
	Raw						Polished					
	y_1	y_2	y_3	x_1	x_2	x_3	y_1	y_2	y_3	x_1	x_2	x_3
1	4	77.25	9.17	14.92	12.17	9.97	-7.27	10.1	0.87	5.24	-0.98	-0.31
2	1.25	54.42	0.5	9	6.08	4.75	-3.3	-0.38	-1.06	6.06	-1.11	1.26
3	0.08	60.75	0	0.83	11.33	1.67	-3.24	3.49	-0.34	-0.88	5.45	-0.60
4	26.08	61.67	3.17	22.08	19.25	9.97	13.85	-4.51	-6.11	11.46	4.455	-1.26
5	8.75	79.67	3.25	6.83	21.5	14.25	-1.35	11.72	-6.2	-3.57	8.76	2.46
6	23.67	32.92	0.08	10.08	6.25	2	19.84	-24.85	-0.76	7.86	-0.14	-0.77
7	13.33	47.42	0.08	3.75	10.75	17.33	7.63	-12.49	-2.44	-0.54	2.63	12.58
8	24.58	35.75	0	1.92	18.58	10.67	16.11	-25.83	-4.65	-4.11	8.38	4.09
9	14.67	45.92	0.25	3.92	3.58	28.67	10.42	-12.26	-1.01	1.28	-3.22	25.48
10	17.42	57.67	16.08	5.17	19.83	16.17	4.08	-9.61	5.73	-6.56	3.94	3.89
11	28.42	31.83	0.5	2.67	8.83	22.42	22.76	-27.76	-2.17	-1.38	0.61	17.82
12	11.08	49.17	0.5	2	23.33	9.5	5.52	-10.34	-2.08	-1.96	15.21	4.99
13	17.83	71.75	1.58	22.25	19.42	9.75	3.90	4.63	-7.44	9.35	3.51	-3.20
14	5.75	75.75	9.67	16.92	6.42	6.42	-4.07	11.99	2.83	8.70	-5.97	-2.35
15	28.58	35	2.17	7.832	17.75	6.17	24.89	-32.79	-4.37	-0.5	10.16	0.20
16	6.42	59.97	22.5	1.33	9	4.67	-0.23	1.59	22.84	-2.45	0.46	0
17	1.17	74.25	15.92	2.17	18.08	1.92	-6.94	12.96	9.30	-4.57	7.76	-5.29
18	2.08	62.33	28.33	4.5	2.58	1.5	-2.58	3.73	26.65	1.44	-4.64	-2.11
19	2.25	61	28	3.67	3	2.33	-2.31	2.51	26.43	0.72	-4.12	-1.16
20	5.33	55.17	0.33	4.92	4.58	4.58	0.96	-3.14	-1.05	2.15	-2.35	1.27
21	2.25	70.83	18.5	4.83	1.92	1.25	-2.93	11.72	16.31	1.26	-5.82	-2.87
22	4.25	57	29.92	3	2.17	1.92	0.22	-0.96	28.88	0.59	-4.42	-1.05
23	2.92	72.92	15.25	2.75	8.92	5.75	-4.24	11.83	11.08	-2.79	-0.8	-0.34
24	4.33	76.83	10	2.58	3.75	4.67	-1.22	17.35	7.44	-1.36	-4.36	0.18
25	3.83	57	0	2.25	1.83	3.75	0.4	-0.37	-0.45	0.43	-4.16	1.38
26	3.833	56.58	0.17	1.83	6.17	1	0.79	-0.4	0.11	0.40	0.56	-0.98

Variance

	Raw						Polished					
Site	y_1	y_2	y_3	x_1	x_2	x_3	y_1	y_2	y_3	x_1	x_2	x_3
1	22.36	40.20	23.42	168.99	136.33	35.72	22.40	44.05	41.19	91.02	22.87	13.33
2	2.02	238.81	0.82	67.27	32.99	27.48	13.2	1.68	4.1	55.82	13.9	7.79
3	0.08	3.48	0	1.79	127.33	6.24	6.79	4.58	1.61	1.72	31.97	1.13
4	84.99	71. 7	7.79	319.90	224.93	55.90	155.05	50.17	10.77	204.09	82.04	37.83
5	71.84	57. 7	5.11	21.97	361	149.84	87.36	185.64	48.96	30.63	101.56	17.82
6	29.33	73.72	0.08	140.08	44.39	3.82	17.77	64.62	3.14	120.62	3.91	0.84
7	55.15	67.17	0.08	8.39	53.66	264.42	64.93	26.11	5.89	10.51	22.59	247.24
8	169.54	206.39	0	5.17	293.17	74.61	112.64	192.58	9.75	21.45	64.59	13.53
9	66.97	80.45	0.20	14.63	16.08	458.24	66.77	61.41	4.99	8.43	13.13	429.56
10	113.17	106.61	86.99	15.61	264.15	158.88	76.06	145.55	117.89	49.26	20.13	26.48
11	138.63	155.79	1	7.88	88. 7	313.72	140.83	123.08	5.29	10.62	9.38	265.04
12	92.27	99.24	1	3.27	375.15	51.55	76.87	82.02	7.11	6.65	163.92	20.68
13	127.61	133.3	3.90	305.48	296.99	79.66	76.71	157.51	58. 2	170.42	33.96	22.17
14	23.66	19.5	29.52	116.99	60.45	18.08	28.31	24.79	21.85	57.84	23.88	3.53
15	274.63	503.64	25.79	45.61	214.57	22.52	117.4	132.65	7.61	6.72	70.93	11.63
16	19.54	166.63	229.36	1.52	106.55	38.79	3.62	95.89	220.34	9.72	16.1	16.07
17	3.424	52.20	46.08	4. 7	373.90	8.08	16.81	26.57	28.04	14.57	88.89	18.47
18	1.72	46.06	53.88	27.73	13.17	3.73	8. 4	18.24	73.61	9.21	41.6	3.92
19	8.022	54.73	38.55	12.42	3.82	2.97	6.91	24.91	63.62	6.47	27.95	4.85
20	29.27	25.06	0.24	23.90	10.27	10.63	21.43	49.31	1.54	9.52	23.40	1.83
21	1.842	25.61	43.18	16. 7	1.90	2.20	6.99	34.19	47.27	2.31	43.2	4.17
22	16.57	50.91	84.81	7.45	5.24	4.08	16.12	46.31	108.7	1.77	28.08	1.98
23	6.99	42.63	41.66	3.66	91.72	19.84	14.39	43.44	53.60	12.3	1.51	0.79
24	12.24	22.15	13.27	4.99	16.02	8.79	4.01	29.93	22.83	2.37	16.81	1.33
25	14.33	13.27	0	4.02	3.24	9.11	6.32	29.39	1.28	0.91	27. 2	2.71
26	3.97	6.99	0.15	2.33	57.79	1.82	2.63	10.13	1.42	2.98	11.53	3.26

(WMO annual summary)

Table 2 Joint Cokriging Predictors

D.(km) Direction	Estimates of Mean						Absolute Bias					
	y_1	y_2	y_3	x_1	x_2	x_3	y_1	y_2	y_3	x_1	x_2	x_3
100	0.95	-6.15	3.21	14.99	-3.05	1.30	2.95	4.15	1.21	2.76	0.66	3.43
500E.	3.63	11.04	5.26	30.50	-1.74	-6.03	1.81	5.4	3.59	1.56	0.14	1.42
500N.	4.81	-1.42	7.68	9.64	-17.77	45.69	11.63	6.38	5.24	11.98	7.50	4.48
500S.E.	9.19	-17.36	38.94	-15.88	15.00	3.13	7.52	4.21	11.73	8.77	4.04	4.73

(WMO annual summary)

Table 3 Joint Cokriging Variance and Lagrange Multipliers

Distance 100km

-170.229	206.271	251.8648			
192.1298	-329.736	-160.445	10.74584	6.809326	-4.57933
14.83763	36.04463	-268.815	-0.42413	16.84432	-12.9198
183.205	-125.946	-63.6878			
-118.543	168.41	6.270001	7.315565	-10.3624	-10.4437
-110.385	15.63556	207.4244	-0.07607	5.226073	-8.71542

Distance 500km East

156.415	-201.658	155.5402			
167.0285	255.4263	-385.201	8.948734	10.37356	1.077626
-2.87781	-14.9159	-4.43353	0.379741	-7.37439	-4.23609
135.4597	284.6396	-88.7732			
-86.3188	-242.895	361.4684	9.120535	2.554407	1.539314
-122.2	-150.302	-149.585	-5.79378	-21.4748	-26.6693

Distance 500km North

-156.415	-201.658	155.5402			
167.0285	255.4263	-385.201	8.948734	10.37356	1.077626
-2.87781	-14.9159	-4.43353	0.379741	-7.37439	-4.23609
135.4597	284.6396	-88.7732			
-86.3188	-242.895	361.4684	9.120535	2.554407	1.539314
-122.2	-150.302	-149.585	-5.79378	-21.4748	-26.6693

Distance 500kmSoutheast

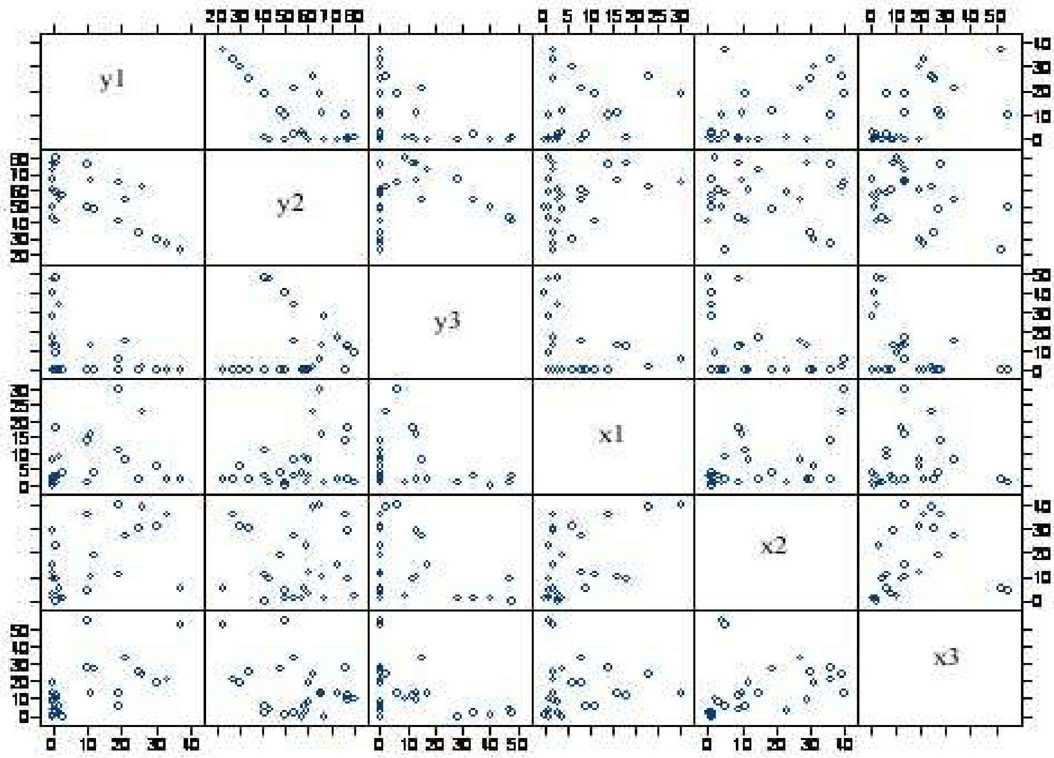
156.415	211.7956	132.2363			
167.0288	-404.434	-132.109	8.948734	10.37356	1.077626
-2.87776	-27.5628	-292.256	0.379741	-7.37439	-4.23609
135.4599	-90.3254	-35.353			
-86.3192	231.4422	-19.6236	9.120535	2.554407	1.539314
-122.2	33.5035	234.4115	-5.79378	-21.4748	-26.6693

(WMO annual summary)

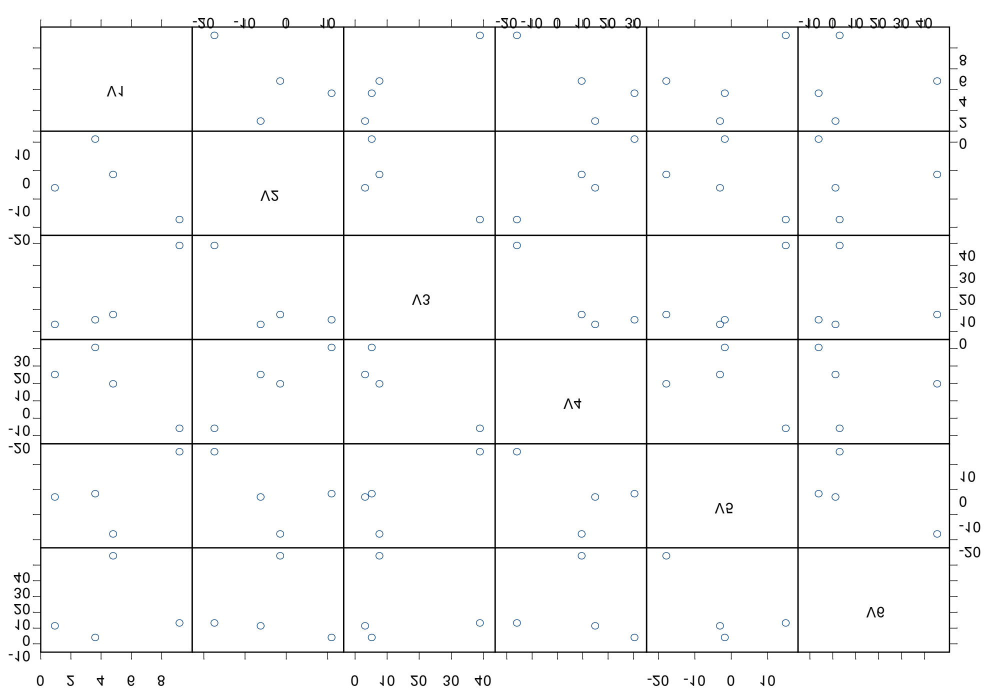
Table 4 Joint Multivariate Regression

Process	y_1	y_2	y_3	x_1	x_2	x_3
Estimates of Mean	12.10	51.97	9.57	10.87	20.96	12.73
Absolute Bias	6.18	13.40	10.23	8.31	10.85	7.29
Mean Square Error	38.19	179.56	04.65	69.06	117.72	53.14

(WMO annual summary)



(WMO annual summary)
Fig1 Coarse Mapping of Monitoring Sites



(WMO annual summary)

Fig.2 Scatter plot matrix for joint prediction