



STOCHASTIC INVENTORY MODEL FOR AMELIORATING ITEMS UNDER SUPPLIER'S TRADE CREDIT POLICY

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ABSTRACT

This paper develops a model to determine an optimal ordering policy for deteriorating items under permissible delay of payment and allowable shortage for future supply uncertainty for two suppliers. In this paper we have introduced the aspect of part payment. A part of the purchased cost is to be paid during the permissible delay period. Spectral theory is used to derive explicit expression for the transition probabilities of a four state continuous time markov chain representing the status of the systems. These probabilities are used to compute the exact form of the average cost expression. We use concepts from renewal reward processes to develop average cost objective function. Optimal solution is obtained using Newton Raphson method in R programming. Finally sensitivity analysis of the varying parameter on the optimal solution is carried out.

KEYWORDS: Future Supply Uncertainty, Partial Payment, Perishable Items, Permissible Delay In Payment And Two Suppliers.

INTRODUCTION

This paper represent practical life situation by assuming that the supplier's market is not monopolistic as competitive spirit in the business is increased especially after induction of multinational companies. We undertake a duopolistic case which can be generalized further. In other words, it is assumed that the inventory manager may place his order with any one of two suppliers. This generalization results is a more difficult problem, however it makes the model more realistic when the manager may receive his supply from more than one source. Here, we assume that the decision maker deals with two suppliers who may be ON or OFF. Here there are three states that correspond to the availability of at least one supplier that is states 0, 1 and 2 where as state 3 denotes the non availability of either of them. Status of both the suppliers is explained as below.

| State | Status of supplier 1 | Status of supplier 2 |
|-------|----------------------|----------------------|
| 0 | ON | ON |
| 1 | ON | OFF |
| 2 | OFF | ON |
| 3 | OFF | OFF |

Here, it is assumed that one may place order to either one of the two suppliers or partly to both when both suppliers are available (i.e. state 0 of the system).

As we know that market is not entirely for whole items but there are items which are of deteriorating nature, so in this paper we have developed inventory model for perishable items. A complete survey of the published literature in mathematical modeling of deteriorating inventory systems is given by Raafat (1991).

In most inventory models it is implicitly assumed that the product to be ordered is always available (i.e. continuous supply availability), that is when an order is placed it is either received immediately or after a deterministic or perhaps random lead time. However, if the product is

purchased from another company (as in the JIT-Just in Time deliveries of parts and components), the supply of the product may sometimes be interrupted due to the supplier's equipment breakdowns, labor strikes or other unpredictable circumstances. Silver (1981) appears to be the first author to discuss the need for models that deal with supplier uncertainty. Articles by Parlar and Berkin (1991) consider the supply uncertainty problem for a class of EOQ model with a single supplier where the availability and unavailability periods constitute an alternating Poisson process. Parlar and Berkin (1991) assume that at any time the decision maker is aware of the availability status of the product although he does not know when the ON (available) and OFF (unavailable) periods will start and end. When the inventory level reaches the reorder point of zero and the status is ON, the order is received; otherwise the decision maker must wait until the product becomes available. Parlar and Perry(1996) developed inventory model for non deteriorating items with future supply uncertainty considering demand rate $d=1$ for two suppliers. Kandpal and Gujarathi (2003,2006) has extended the model of Parlar & Perry(1996) by considering demand rate greater than one and for deteriorating items for single supplier. Kandpal and Tinani (2010) developed inventory model for deteriorating items with future supply uncertainty and permissible delay in payment for two suppliers.

In today's business transactions it is more and more common to see that the customers are allowed some grace period before settling the account with the supplier. This provides an advantage to the customers, due to the fact that they do not have to pay the supplier immediately after receiving the product but instead, can defer their payment until the end of the allowed period. The customer pays no interest during the fixed period, but if the payment is delayed beyond that period, interest will be charged. The customer can start to accumulate revenues on the sale or

use of the product, and earn interest on that revenue. So, it is to the advantage of the customer to defer the payment to the supplier until the end of the period. Shortages are very important, especially in a model that considers delay in payment due to the fact that shortages can affect the quantity ordered to benefit from the delay in payment. Goyal (1985) has studied an EOQ system with deterministic demand and permissible delay in payments which was reinvestigated by Chand and Ward (1987). Shah (1993) developed model for deteriorating items when delay in payments is permissible by assuming deterministic demand. Aggarwal and Jaggi (1995) developed a model to determine the optimum order quantity for deteriorating items under a permissible delay in payment. In this paper, we have introduced the aspect of part payment. It is common practice that an installment of payments is made during the period of the admitted delay in payment. The part to be paid and the time at which it is to be paid are mutually settled between the supplier and the buyer at the time of purchase of goods. Kandpal and Tinani (2011) developed perishable-inventory model under inflation and delay in payment allowing partial payment for single supplier.

NOTATIONS, ASSUMPTIONS AND MODEL

The inventory model here is developed on the basis of the following assumptions.

- (a) Demand rate d is deterministic and it is $d > 1$.
- (b) We define X_i and Y_i be the random variables corresponding to the length of ON and OFF period respectively for i^{th} supplier where $i=1, 2$. We specifically assume that $X_i \sim \exp(\lambda_i)$ and $Y_i \sim \exp(\mu_i)$. Further X_i and Y_i are independently distributed.
- (c) Ordering cost is Rs. k /order.
- (d) Holding cost is Rs. h /unit/unit time.
- (e) Shortage cost is Rs. π /unit.
- (f) Time dependent part of the backorder cost is Rs. $\hat{\pi}$ /unit/time.
- (g) q_i = order upto level $i=0, 1, 2$
- (h) r =reorder upto level; q_i and r are decision variables.
- (i) θ is the rate of deterioration which is constant fraction of on hand inventory. The deteriorated units can neither be replaced nor repaired during cycle period.
- (j) Purchase cost is Rs. c /unit.
- (k) T_{li} is the time allowed by i^{th} supplier where $i=1, 2$ at which α_i ($0 < \alpha_i < 1$) fraction of total amount has to be paid to the i^{th} supplier where $i=1, 2$.
- (l) T_i ($T_i > T_{li}$) is the time at which remaining amount has to be cleared.
- (m) T_{00} is the expected cycle time. T_{li} and T_i are known constants and T_{00} is a decision variable.
- (n) Ie_i =Interest rate earned when purchase made from i^{th} supplier where $i=1, 2$
 Ic_i =Interest rate charged by i^{th} supplier where $i=1, 2$.

- (o) U_i and V_i are indicator variables for i^{th} supplier where $i=1, 2$

$$U_1 = 0 \quad \text{if part payment is done at } T_{11} \\ = 1 \quad \text{otherwise and}$$

$$U_2 = 0 \quad \text{if part payment is done at } T_{12} \\ = 1 \quad \text{otherwise}$$

And

$$V_1 = 0 \quad \text{if the balanced amount is cleared at } T_1 \\ = 1 \quad \text{otherwise and}$$

$$V_2 = 0 \quad \text{if the balanced amount is cleared at } T \\ = 1 \quad \text{otherwise}$$

- (p) $A(q_i, r, \theta)$ = Actual cost incurred when the inventory drops to r and state is ON for the i^{th} supplier, $i=1, 2$.

In this paper, we assume that supplier allows a fixed period T_{li} during which α_i fraction of total amount has to be paid and remaining amount i.e. $(1 - \alpha_i)$ fraction has to be cleared upto time T_i . Hence upto time period T_{li} no interest is charged for α_i fraction, but beyond that period, interest will be charged upon not doing promised payment of α_i fraction. Similarly for $(1 - \alpha_i)$ fraction no interest will be charged up to time period T_i but beyond that period interest will be charged. However, customer can sell the goods and earn interest on the sales revenue during the period of admissible delay.

Interest earned and interest charged is as follows.

- (i) Interest earned on the entire amount up to time period T_{li} is $dcT_{li}T_{00}Ie_i$
- (ii) Interest earned on $(1 - \alpha_i)$ fraction during the period $(T_i - T_{li})$ is $(1 - \alpha_i)d c(T_i - T_{li})T_{00}Ie_i$
- (iii) If part payment is not done at T_{li} then interest will be earned over α_i fraction for period $(T_i - T_{li})$ but interest will also be charged for α_i fraction for $(T_i - T_{li})$ period.

$$\text{Interest earned} = d c \alpha_i T_{00} (T_i - T_{li}) Ie_i$$

$$\text{Interest charged} = d c \alpha_i T_{00} (T_i - T_{li}) Ic_i$$

To discourage not doing promised payment, we assume that Ic_i is quite larger than Ie_i .

- (iv) Interest earned over the amount $d c T_{00} T_{li} Ie_i$ over the period $(T_i - T_{li})$ is $d c T_{00} T_{li} Ie_i (T_i - T_{li}) Ie_i$
- (v) If the remaining amount is not cleared at T_i then interest will be earned for the period $(T_{00} - T_i)$ for $(1 - \alpha_i)$ fraction simultaneously interest will be charged on the same amount for the same period.

Interest earned= $d c (1 - \alpha_i) T_{00} (T_{00} - T_i) I e_i$ and

Interest charged= $d c (1 - \alpha_i) T_{00} (T_{00} - T_i) I c_i$

Total interest earned = $d c T_{li} T_{00} I e_i +$

$(1 - \alpha_i) d c (T_i - T_{li}) T_{00} I e_i +$

$d c \alpha_i T_{00} (T_i - T_{li}) I e_i +$

$d c T_{00} T_{li} I e_i (T_i - T_{li}) I e_i +$

$V_i [d c (1 - \alpha_i) T_{00} (T_{00} - T_i) I e_i +$

$d c T_{li} T_{00} I e_i (T_i - T_{li}) I e_i (T_{00} - T_i) I e_i +$

$d c T_{li} T_{00} I e_i (T_{00} - T_i) I e_i +$

$d c (1 - \alpha_i) T_{00} (T_i - T_{li}) I e_i (T_{00} - T_i) I e_i +$

$\{ d c \alpha_i T_{00} I e_i (T_i - T_{li}) I e_i -$

$d c \alpha_i T_{00} I c_i (T_{00} - T_i) \}]$

Total Interest charged= $d c \alpha_i T_{00} (T_i - T_{li}) I c_i +$

$V_i [d c (1 - \alpha_i) T_{00} (T_{00} - T_i) I c_i]$

- (v) If the remaining amount is not cleared at T_i then interest will be earned for the period $(T_{00} - T_i)$ for $(1 - \alpha_i)$ fraction simultaneously interest will be charged on the same amount for the same period.

Interest earned= $d c (1 - \alpha_i) T_{00} (T_{00} - T_i) I e_i$ and

Interest charged= $d c (1 - \alpha_i) T_{00} (T_{00} - T_i) I c_i$

Total interest earned = $d c T_{li} T_{00} I e_i +$

$(1 - \alpha_i) d c (T_i - T_{li}) T_{00} I e_i +$

$d c \alpha_i T_{00} (T_i - T_{li}) I e_i +$

$d c T_{00} T_{li} I e_i (T_i - T_{li}) I e_i +$

$V_i [d c (1 - \alpha_i) T_{00} (T_{00} - T_i) I e_i +$

$d c T_{li} T_{00} I e_i (T_i - T_{li}) I e_i (T_{00} - T_i) I e_i +$

$d c T_{li} T_{00} I e_i (T_{00} - T_i) I e_i +$

$d c (1 - \alpha_i) T_{00} (T_i - T_{li}) I e_i (T_{00} - T_i) I e_i +$

$\{ d c \alpha_i T_{00} I e_i (T_i - T_{li}) I e_i -$

$d c \alpha_i T_{00} I c_i (T_{00} - T_i) \}]$

Total Interest charged= $d c \alpha_i T_{00} (T_i - T_{li}) I c_i + V_i [$

$d c (1 - \alpha_i) T_{00} (T_{00} - T_i) I c_i]$

Total interest earned and charged is as follows.

$d c T_{li} T_{00} I e_i + (1 - \alpha_i) d c (T_i - T_{li}) T_{00} I e_i + \{$

$d c \alpha_i T_{00} (T_i - T_{li}) I e_i - d c \alpha_i T_{00} (T_i - T_{li}) I c_i \}$

$+ d c T_{00} T_{li} I e_i (T_i - T_{li}) I e_i + V_i [$

$d c (1 - \alpha_i) T_{00} (T_{00} - T_i) I e_i + d c T_{li} T_{00} I e_i$

$(T_i - T_{li}) I e_i (T_{00} - T_i) I e_i$

$+ d c T_{li} T_{00} I e_i (T_{00} - T_i) I e_i +$

$d c (1 - \alpha_i) T_{00} (T_i - T_{li}) I e_i (T_{00} - T_i) I e_i + \{$

$d c \alpha_i T_{00} I e_i (T_i - T_{li}) I e_i$

$- d c \alpha_i T_{00} I c_i (T_{00} - T_i)$

$d c (1 - \alpha_i) T_{00} (T_{00} - T_i) I c_i]$

The policy we have chosen is denoted by (q_0, q_1, q_2, r) . An order is placed for q_i units $i=0, 1, 2$, whenever, inventory drops to the reorder point r and the state found is $i=0, 1, 2$. When both suppliers are available, q_0 is the total ordered from either one or both suppliers. If the process is found in state 3 that is both the suppliers are not available nothing can be ordered in which case the buffer stock of r units is reduced. If the process stays in state 3 for longer time then the shortages start accumulating at rate of d units/time. When the process leaves state 3 and supplier becomes available, enough units are ordered to increase the inventory to $q_i + r$ units where $i=0, 1, 2$.

$A(q_i, r, \theta) =$ (cost of ordering) + (cost of holding inventory) + (cost of item that deteriorate during a single interval that starts with an inventory of $(q_i + r)$ units and ends with r units);

$$A(q_i, r, \theta) = k + \frac{1}{2} \cdot \frac{h q_i^2}{(d + \theta)} + \frac{h r q_i}{(d + \theta)} + \frac{\theta c q_i}{(d + \theta)}$$

$i=0, 1, 2$

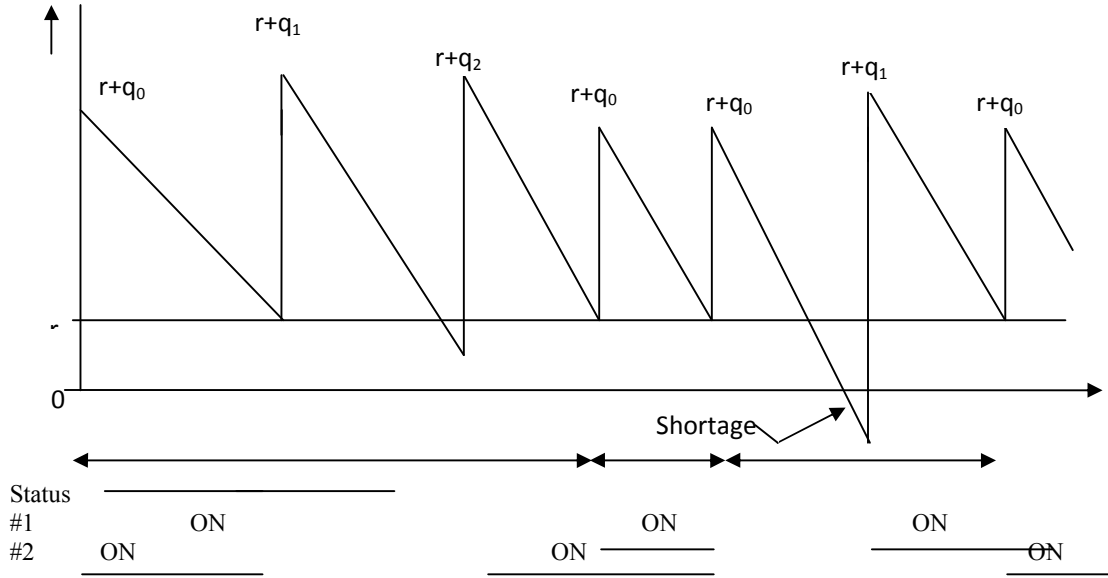
$C_{00} = E$ (cost per cycle); and $T_{00} = E$ (length of a cycle);

$P_{ij}(t) = P$ (Being in state j at time t /starting in state i at time 0, $i, j=0, 1, 2, 3$);

$p_i =$ long run probabilities, $i=0, 1, 2, 3$

3. OPTIMAL POLICY DECISION FOR THE MODEL

For calculation of average cost objective function, we need to identify the cycles. Below given figure gives us the idea about cycles and their identification



Referring to Figure 3.1, we see that the cycles of this process start when the inventory goes up to a level of q_0+r units. Once the cycle is identified, we construct the average cost objective function as a ratio of the expected cost per cycle to the expected cycle length.

$$\text{i.e. } Ac(q_0, q_1, q_2, r) = \frac{C_{00}}{T_{00}}$$

where,

C_{00} =E (cost per cycle) and T_{00} =E (length of a cycle)

Analysis of the average cost function requires the exact determination of the transition probabilities $P_{ij}(t)$, $i, j=0, 1, 2, 3$ for the four state continuous time markov chain (CTMC). The solution is provided in the following lemma.

Lemma 3.1:- Let $P(t) = [P_{ij}(t)]$ $t \geq 0$, $i, j=0, 1, 2, 3$ be 4×4 matrix of transition functions for the continuous time markov chain(CTMC). The exact transient solution is given as $P(t) = UD(t)U^{-1}$.where,

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -\mu_2/\lambda_2 & -\mu_2/\lambda_2 \\ 1 & -\mu_1/\lambda_1 & 1 & -\mu_1/\lambda_1 \\ 1 & -\mu_1/\lambda_1 & -\mu_2/\lambda_2 & \mu_1\mu_2/\lambda_1\lambda_2 \end{bmatrix}$$

$$D(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-(\lambda_1+\mu_1)t} & 0 & 0 \\ 0 & 0 & e^{-(\lambda_2+\mu_2)t} & 0 \\ 0 & 0 & 0 & e^{-(\lambda_1+\mu_1+\lambda_2+\mu_2)t} \end{bmatrix}$$

$$U^{-1} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} \begin{bmatrix} \mu_1\mu_2 & \lambda_2\mu_1 & \lambda_1\mu_2 & \lambda_1\lambda_2 \\ \lambda_1\mu_2 & \lambda_1\lambda_2 & -\lambda_1\mu_2 & -\lambda_1\lambda_2 \\ \lambda_2\mu_1 & -\lambda_2\mu_1 & \lambda_1\lambda_2 & -\lambda_1\lambda_2 \\ \lambda_1\lambda_2 & -\lambda_1\lambda_2 & -\lambda_1\lambda_2 & \lambda_1\lambda_2 \end{bmatrix}$$

Proof:- For proof refer kandpal and Tinani(2010)

Corollary 3.2:- The long run probabilities

$$P_j = \lim_{t \rightarrow \infty} P_{ij}(t) \text{ are}$$

$$[P_0, P_1, P_2, P_3] = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} [\mu_1\mu_2, \lambda_2\mu_1, \lambda_1\mu_2, \lambda_1\lambda_2]$$

Proof:- As $t \rightarrow \infty$, we have

$$\lim_{t \rightarrow \infty} D(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and hence the proof.

Define C_{i0} =E(cost incurred to the beginning of the next cycle from the time when inventory drops to r at state

$i=0, 1, 2, 3$ and q_i units are ordered if $i=0, 1$ or 2)

Lemma 3.3:- C_{i0} is given by

$$C_{i0} = P_{i0} \left(\frac{q_i}{d+\theta} \right) A(q_i, r) + \sum_{j=1}^3 P_{ij} \left(\frac{q_i}{d+\theta} \right) [A(q_i, r) + C_{j0}] \quad i = 0, 1, 2 \quad (3.3.1)$$

$$C_{30} = \bar{C} + \sum_{i=1}^2 \rho_i C_{i0} \quad (3.3.2)$$

Where $\rho_i = \frac{\mu_i}{\delta}$ with $\delta = \mu_1 + \mu_2$ and

$$\bar{C} = \frac{e^{-\frac{\delta r}{(d+\theta)}}}{\delta^2} \left[h e^{\frac{\delta r}{(d+\theta)}} (\delta r - (d+\theta)) + (\pi \delta d + h(d+\theta) + \pi) - \theta c \delta \right] + \frac{\theta c}{\delta}$$

Proof:- For proof refer Kandpal and Tinani(2010)

Lemma 3.4:- The exact expression for C_{00} is

$$C_{00} = A_0 + P_{01} C_{10} + P_{02} C_{20} + P_{03} (\bar{C} + \rho_1 C_{10} + \rho_2 C_{20}) \quad (3.4.1)$$

where the pair $[C_{10}, C_{20}]$ solves the system

$$\begin{bmatrix} 1 - P_{11} - P_{13} \rho_1 & -(P_{12} + P_{13} \rho_2) \\ -(P_{21} + P_{23} \rho_1) & 1 - P_{22} - P_{23} \rho_2 \end{bmatrix} \begin{bmatrix} C_{10} \\ C_{20} \end{bmatrix} = \begin{bmatrix} A_1 + P_{13} \bar{C} \\ A_2 + P_{23} \bar{C} \end{bmatrix} \quad (3.4.2)$$

Proof:- Rearranging the linear system of four equations in lemma(3.3) in matrix form gives

$$\begin{bmatrix} 1 & -P_{01} & -P_{02} & -P_{03} \\ 0 & 1 - P_{11} & -P_{12} & -P_{13} \\ 0 & -P_{21} & 1 - P_{22} & -P_{23} \\ 0 & -\rho_1 & -\rho_2 & 1 \end{bmatrix} \begin{bmatrix} C_{00} \\ C_{10} \\ C_{20} \\ C_{30} \end{bmatrix} = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ \bar{C} \end{bmatrix} \quad (3.4.3)$$

We have $C_{30} = \bar{C} + \rho_1 C_{10} + \rho_2 C_{20}$ from the last row of the system. Substituting this result in rows two and three and rearranging gives the system in (3.4.2), with (C_{10}, C_{20})

From the first row of (3.4.3) we obtain

$$C_{00} = A_0 + \sum_{j=1}^3 P_{0j} C_{j0}.$$

Hence the proof.

Define, $T_{i0} = E[\text{Time to the beginning of the next cycle from the time when inventory drops to } r \text{ at state } i=0, 1, 2, 3 \text{ and } q_i \text{ units are ordered if } i=0, 1, 2]$

Lemma 3.5:- Expected cycle length is given by

$$T_{i0} = P_{i0} \left(\frac{q_i}{d+\theta} \right) \frac{q_i}{d+\theta} + \sum_{j=1}^3 P_{ij} \left(\frac{q_i}{d+\theta} \right) \left[\frac{q_i}{d+\theta} + T_{j0} \right] \quad i = 0, 1, 2$$

$$T_{30} = \bar{T} + \sum_{i=1}^2 \rho_i T_{i0}$$

where $\bar{T} = \frac{1}{\mu_1 + \mu_2}$ is the expected time from the time inventory drops to r until either supplier 1 or 2 becomes available.

Lemma 3.6:- The exact expression for T_{00} is

$$T_{00} = \frac{q_0}{d+\theta} + P_{01} T_{10} + P_{02} T_{20} + P_{03} (\bar{T} + \rho_1 T_{10} + \rho_2 T_{20})$$

where the pair $[T_{10}, T_{20}]$ solves the system.

$$\begin{bmatrix} 1 - P_{11} - P_{13}\rho_1 & -(P_{12} + P_{13}\rho_2) \\ -(P_{21} + P_{23}\rho_1) & (1 - P_{22} - P_{23}\rho_2) \end{bmatrix} \begin{bmatrix} T_{10} \\ T_{20} \end{bmatrix} = \begin{bmatrix} q_1 + P_{13}\bar{T} \\ q_2 + P_{23}\bar{T} \end{bmatrix}$$

The proof of the above two lemmas i.e. (3.5) and (3.6) are very similar to lemma (3.3) and (3.4).

Theorem 3.7:- The Average cost objective function for two suppliers under permissible delay in payments allowing partial

payment is given by $Ac = \frac{C_{00}}{T_{00}}$

C_{00} is given by

$$\begin{aligned} C_{00} = & A(q_0, r) + P_{01} \{ C_{10} - dcT_{00}T_{11}Ie_1 - (1 - \alpha_1)dcT_{00}(T_1 - T_{11})Ie_1 - U_1dc\alpha_1T_{00}(T_1 - T_{11})Ie_1 \\ & + U_1dc\alpha_1T_{00}(T_1 - T_{11})Ic_1 - dcT_{00}T_{11}Ie_1(T_1 - T_{11})Ie_1 \\ & - V_1 \left[(1 - \alpha_1)dcT_{00}(T_{00} - T_1)Ie_1 + dcT_{00}T_{11}Ie_1(T_1 - T_{11})Ie_1(T_{00} - T_1)Ie_1 \right. \\ & \left. + dcT_{00}T_{11}Ie_1(T_{00} - T_1)Ie_1 + (1 - \alpha_1)dcT_{00}(T_1 - T_{11})Ie_1(T_{00} - T_1)Ie_1 \right] \\ & - V_1 \left[U_1 \{ dc\alpha_1T_{00}Ie_1(T_1 - T_{11})(T_{00} - T_1)Ie_1 \} \right] \\ & + V_1 \left[U_1 \{ dc\alpha_1T_{00}Ic_1(T_{00} - T_1) + (1 - \alpha_1)dcT_{00}Ic_1(T_{00} - T_1) \} \right] \} \\ & + P_{02} \{ C_{20} - dcT_{00}T_{12}Ie_2 - (1 - \alpha_2)dcT_{00}(T_2 - T_{12})Ie_2 - U_2dc\alpha_2T_{00}(T_2 - T_{12})Ie_2 \\ & + U_2dc\alpha_2T_{00}(T_2 - T_{12})Ic_2 - dcT_{00}T_{12}Ie_1(T_2 - T_{12})Ie_2 \\ & - V_2 \left[(1 - \alpha_2)dcT_{00}(T_{00} - T_2)Ie_2 + dcT_{00}T_{12}Ie_2(T_2 - T_{12})Ie_2(T_{00} - T_2)Ie_2 \right. \\ & \left. + dcT_{00}T_{12}Ie_2(T_{00} - T_2)Ie_2 + (1 - \alpha_2)dcT_{00}(T_2 - T_{12})Ie_2(T_{00} - T_2)Ie_2 \right] \\ & - V_2 \left[U_2 \{ dc\alpha_2T_{00}Ie_2(T_2 - T_{12})(T_{00} - T_2)Ie_2 \} \right] \\ & + V_2 \left[U_2 \{ dc\alpha_2T_{00}Ic_2(T_{00} - T_2) + (1 - \alpha_2)dcT_{00}Ic_2(T_{00} - T_2) \} \right] \} \\ & + P_{03} \left\{ \bar{C} + \rho_1 \left(\begin{aligned} & C_{10} - dcT_{00}T_{11}Ie_1 - (1 - \alpha_1)dcT_{00}(T_1 - T_{11})Ie_1 - U_1dc\alpha_1T_{00}(T_1 - T_{11})Ie_1 \\ & + U_1dc\alpha_1T_{00}(T_1 - T_{11})Ic_1 - dcT_{00}T_{11}Ie_1(T_1 - T_{11})Ie_1 \\ & - V_1 \left[(1 - \alpha_1)dcT_{00}(T_{00} - T_1)Ie_1 + dcT_{00}T_{11}Ie_1(T_1 - T_{11})Ie_1(T_{00} - T_1)Ie_1 \right. \\ & \left. + dcT_{00}T_{11}Ie_1(T_{00} - T_1)Ie_1 + (1 - \alpha_1)dcT_{00}(T_1 - T_{11})Ie_1(T_{00} - T_1)Ie_1 \right] \\ & - V_1 \left[U_1 \{ dc\alpha_1T_{00}Ie_1(T_1 - T_{11})(T_{00} - T_1)Ie_1 \} \right] \\ & + V_1 \left[U_1 \{ dc\alpha_1T_{00}Ic_1(T_{00} - T_1) + (1 - \alpha_1)dcT_{00}Ic_1(T_{00} - T_1) \} \right] \end{aligned} \right) \right. \\ & + \rho_2 \left(\begin{aligned} & C_{20} - dcT_{00}T_{12}Ie_2 - (1 - \alpha_2)dcT_{00}(T_2 - T_{12})Ie_2 - U_2dc\alpha_2T_{00}(T_2 - T_{12})Ie_2 \\ & + U_2dc\alpha_2T_{00}(T_2 - T_{12})Ic_2 - dcT_{00}T_{12}Ie_1(T_2 - T_{12})Ie_2 \\ & - V_2 \left[(1 - \alpha_2)dcT_{00}(T_{00} - T_2)Ie_2 + dcT_{00}T_{12}Ie_2(T_2 - T_{12})Ie_2(T_{00} - T_2)Ie_2 \right. \\ & \left. + dcT_{00}T_{12}Ie_2(T_{00} - T_2)Ie_2 + (1 - \alpha_2)dcT_{00}(T_2 - T_{12})Ie_2(T_{00} - T_2)Ie_2 \right] \\ & - V_2 \left[U_2 \{ dc\alpha_2T_{00}Ie_2(T_2 - T_{12})(T_{00} - T_2)Ie_2 \} \right] \\ & + V_2 \left[U_2 \{ dc\alpha_2T_{00}Ic_2(T_{00} - T_2) + (1 - \alpha_2)dcT_{00}Ic_2(T_{00} - T_2) \} \right] \end{aligned} \right) \} \end{aligned}$$

$$\text{And } T_{00} = \frac{q_0}{d + \theta} + P_{01}T_{10} + P_{02}T_{20} + P_{03}(\bar{T} + \rho_1T_{10} + \rho_2T_{20})$$

Proof:- Proof follows using Renewal reward theorem(RRT). The optimal solution for q_0, q_1, q_2 and r is obtained by using Newton Raphson method in R programming

4. NUMERICAL ANALYSIS

There are sixteen different patterns of payments, some of them we consider here.

1. $U_i=0$ and $V_i=0$ where $i=1, 2$ that is promise of doing part payment at time T_{1i} and Clearing the remaining amount at time T_i both are satisfied, the time period given by i^{th} supplier where $i=1, 2$.
2. $U_i=0$ and $V_i=1$ where $i=1, 2$ that is promise of doing part payment at time T_{1i} is satisfied but remaining amount is not cleared at time T_i , the time period given by i^{th} supplier where $i=1, 2$.
3. $U_i=1$ and $V_i=0$ where $i=1, 2$ that is promise of doing part payment at time T_{1i} is not satisfied but all the amount is cleared at time T_i , the time period given by i^{th} supplier where $i=1, 2$.

| (U_1, U_2, V_1, V_2) | q_0 | q_1 | q_2 | r | Ac |
|------------------------|--------|--------|--------|--------|--------|
| (0, 0, 0, 0) | 3.2890 | 30.178 | 29.580 | 0.7459 | 6.4060 |
| (0, 0, 1, 1) | 2.9496 | 29.824 | 29.144 | 0.6646 | 6.5076 |
| (1, 1, 0, 0) | 3.3466 | 30.154 | 29.561 | 0.7667 | 6.3732 |
| (1, 0, 0, 0) | 3.3140 | 30.168 | 29.572 | 0.7548 | 6.3926 |
| (0, 0, 1, 0) | 3.1550 | 30.040 | 29.411 | 0.7147 | 6.4431 |
| (0, 0, 0, 1) | 3.0487 | 29.914 | 29.257 | 0.6909 | 6.4753 |
| (0, 1, 0, 0) | 3.3220 | 30.164 | 29.569 | 0.7578 | 6.3867 |

From this we conclude that the cost is minimum if part payment is not done at T_{1i} but account is cleared at T_i and the cost is maximum if part payment is done at T_{1i} but account is not cleared at T_i , this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period.

5. SENSITIVITY ANALYSIS

In this section we verify the results by a numerical example. We assume that

$k=\text{Rs. } 5/\text{order}$, $c=\text{Rs. } 1/\text{unit}$, $d=20/\text{units}$, $\theta=4$, $h=\text{Rs. } 5/\text{unit/time}$, $\pi=\text{Rs. } 350/\text{unit}$, $\hat{\pi}=\text{Rs. } 25/\text{unit/time}$, $\alpha_1=0.5$, $\alpha_2=0.6$, $I_{c1}=0.11$, $I_{e1}=0.02$, $I_{c2}=0.13$, $I_{e2}=0.04$, $T_{11}=0.6$, $T_{12}=0.8$, $T_1=0.9$, $T_2=1.1$, $\lambda_1=0.58$, $\lambda_2=0.45$, $\mu_1=3.4$, $\mu_2=2.5$.

The last four parameters indicate that the expected lengths of the ON and OFF periods for first and second supplier are $1/\lambda_1=1.72413794$, $1/\lambda_2=2.2222$, $1/\mu_1=.2941176$ and $1/\mu_2=.4$ respectively. The long run probabilities are obtained as $p_0=0.7239588$, $p_1=0.1303126$, $p_2=0.1234989$ and $p_3=0.02222979$. The optimal solution for the above numerical example based on the seven patterns of payment is obtained as

We study below in the Sensitivity analysis, the effect of change in the parameter on the following three patterns of payment.

- (i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 keeping other parameter values fixed where $U_i=0$ and $V_i=0$ where $i=1, 2$. We resolve the problem to find optimal values of q_0 , q_1 , q_2 , r and Ac . The optimal values of q_0 , q_1 , q_2 and Ac plotted in Fig.5.1

| μ_1 | q_0 | q_1 | q_2 | r | Ac |
|---------|--------|--------|--------|--------|--------|
| 2.4 | 3.1989 | 31.742 | 31.195 | 1.6671 | 6.8755 |
| 3 | 3.2500 | 30.764 | 30.153 | 1.0378 | 6.5665 |
| 3.4 | 3.2890 | 30.178 | 29.580 | 0.7459 | 6.4060 |
| 4.4 | 3.3954 | 28.947 | 28.514 | 0.2633 | 6.1107 |
| 4.8 | 3.4374 | 28.539 | 28.201 | 0.1312 | 6.0228 |

Table 5.1 Sensitivity Analysis Table by varying the parameter values of μ_1 when patterns of payment is ($U_1=0$, $U_2=0$, $V_1=0$, $V_2=0$)

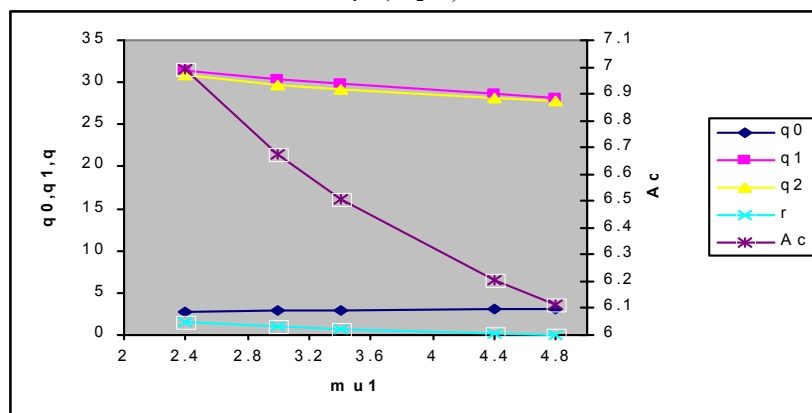


Fig. 5.1 Sensitivity analysis graph for μ_1

We see from 5.1 table and figure that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases the value of q_1 and r decreases which result in decrease in average cost.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity

analysis by varying the value μ_1 keeping other parameter values fixed where $U_i=0$ and $V_i=1$ where $i=1, 2$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and Ac . The optimal values of q_0, q_1, q_2 and Ac plotted in Fig.5.2.

| μ_1 | q_0 | q_1 | q_2 | r | Ac |
|---------|--------|--------|--------|--------|--------|
| 2.4 | 2.7897 | 31.375 | 30.797 | 1.5588 | 6.9929 |
| 3 | 2.8875 | 30.408 | 29.729 | 0.9462 | 6.6734 |
| 3.4 | 2.9496 | 29.824 | 29.144 | 0.6646 | 6.5076 |
| 4.4 | 3.0940 | 28.586 | 28.058 | 0.2048 | 6.2026 |
| 4.8 | 3.1492 | 28.174 | 27.741 | 0.0805 | 6.1116 |

Table 5.2 Sensitivity Analysis Table by varying the parameter values of μ_1
When patterns of payment is ($U_1=0, U_2=0, V_1=1, V_2=1$)

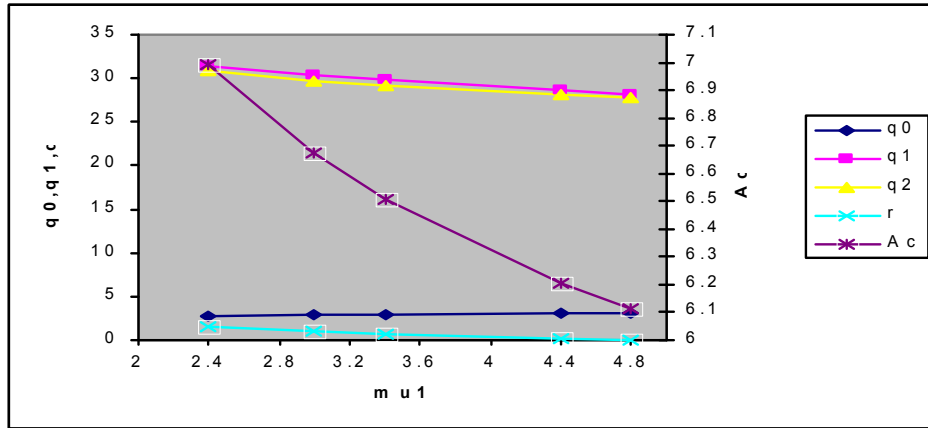


Fig. 5.2 Sensitivity analysis graph for μ_1

We see from 5.2 table and figure that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases the value of q_1 and r decreases which result in decrease in average cost.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity

analysis by varying the value μ_1 keeping other parameter values fixed where $U_i=1$ and $V_i=0$ where $i=1, 2$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and Ac . The optimal values of q_0, q_1, q_2 and Ac plotted in Fig.5.3.

| μ_1 | q_0 | q_1 | q_2 | R | Ac |
|---------|--------|--------|--------|--------|--------|
| 2.4 | 3.2628 | 31.715 | 31.168 | 1.6931 | 6.8421 |
| 3 | 3.3091 | 30.739 | 30.131 | 1.0607 | 6.5335 |
| 3.4 | 3.3466 | 30.154 | 29.561 | 0.7667 | 6.3732 |
| 4.4 | 3.4478 | 28.927 | 28.500 | 0.2796 | 6.0783 |
| 4.8 | 3.4883 | 28.521 | 28.189 | 0.1459 | 5.9900 |

Table 5.3 Sensitivity Analysis Table by varying the parameter values of μ_1
When patterns of payment is ($U_1=1, U_2=1, V_1=0, V_2=0$)

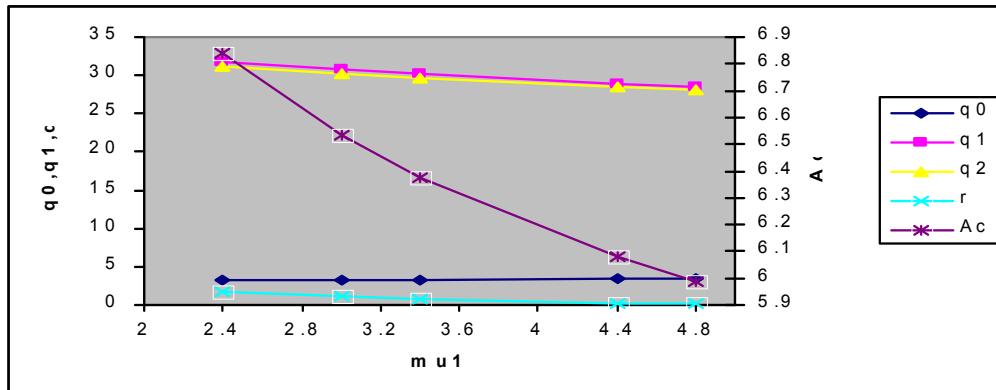


Fig. 5.3. Sensitivity analysis graph for μ_1

We see from 5.3 table and figure that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases the value of q_1 and r decreases which result in decrease in average cost.

CONCLUSION

From the above sensitivity analysis we conclude that cost is minimum if part payment is not done at T_{1i} but account is cleared at T_i and the cost is maximum if part payment is done at T_{1i} but account is not cleared at T_i , this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period.

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