



## PERFORMANCE ANALYSIS OF MIMO SYSTEM OVER OPTIMUM POWER AND RATE ADAPTATION TECHNIQUE FOR TWDP FADING CHANNEL

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### ABSTRACT

This paper investigates the performance of an uncorrelated MIMO system over Optimum Power and Rate Adaptation Technique (OPRA) considering Two Wave Diffuse Power [TWDP] fading channels. Here well known water filling algorithm is used for MIMO system Design. The use of antenna arrays at both sides of the wireless communication link (MIMO systems) can increase the channel capacity than the OPRA Technique. Here the channel capacity is studied for arbitrary no. of antennas and a block of slow frequency non selective channel is assumed. The effect of convenient parameter ( $\Delta$ ) of TWDP fading model, on the capacity analysis of MIMO channels and OPRA policy is also investigate.

**KEYWORDS:** Channel capacity, TWDP fading channel, MIMO Channels, OPRA, Spatial multiplexing.

### INTRODUCTION

In wireless communication system the use of multiple no. of transmitting and receiving antennas [MIMO system] increase the system performance than the traditional single input single output [SISO] system without increasing the available frequency bandwidth. Optimum Power and Rate Adaptation one of the well known adaptation techniques generally used in SISO and SIMO systems for performance enhancement. On the other hand in mobile radio channels the propagation of transmitted signal is characterized by various effects including fading and shadowing. There are different distributions exist that describe the statistics of the mobile radio channels. For example Rayleigh, Rice, Nakagami- $m$ , Weibull and Hoyt are the well known distributions for short term signal fluctuations. However, in sometimes, none of the above distributions is, able to characterize the narrowband fading measurement [1]. So a new fading model has been introduced which provide more flexibility to the fading model [2]. TWDP [Two Wave Diffused Power] is based on this new analytical model. TWDP fading model consist of two specular multipath components in the presence of diffusely propagating waves. This fading model can give a better representation of a real-world fading environment. TWDP can represent the Rayleigh and Rician Fading model as a special case. Still a few works have been done on this fading model till now [3]-[6].

In [3], bit error rate performance expressions for an uncoded binary phase-shift keying (BPSK) system is derived using alternate expression of Gaussian Q function [7] and for maximal ratio combining (MRC) system, performance of BPSK is presented in [5]. Average bit error rate (ABER) performance of Gray coded QAM signaling in TWDP environment is presented in [4] using the cumulative distribution function of TWDP fading and that for MRC system is derived in [6].

Performance analysis over fading channel is very much important for wireless system design. Here the performance analysis of MIMO system over OPRA technique is done by means of capacity analysis. MIMO system can improve spectrum efficiency and signal quality much better than the traditional single antenna system even after using the adaptation policies too. On the other hand if the transmitter side has the knowledge about the channel then the water filling algorithm can be implemented to further increase the system performance [8]. Few works have been published on the MIMO channel capacity over various fading channels [8]- [15] and provide useful analytical support to the system design engineers. But, for TWDP fading capacity analysis for MIMO channels is not available in the literature. This generates a motive to analysis the capacity for MIMO channels over TWDP fading channels.

The rest of the paper is organized as follows. In Section II, the channel has been elaborated. The MIMO system model has been studied in section III. The MIMO channel capacity is describe in section IV. In section V we discuss the OPRA technique and capacity analysis over TWDP fading channel. In section VI the simulated and analytical results and discussion have been given. The paper is concluded in section VII.

### TWDP CHANNELS

The channel has been assumed to be slow (narrow band), frequency non selective with TWDP fading statistics which describes the local area fading of two specular multipath components in the presence of other diffusely propagating waves. Which can be shown by the equation [2]

$$\phi = \sum_{i=1}^N v_i \exp(j\phi_i) + v_{diff} \quad \dots\dots\dots (1)$$

Where,  $\hat{v}$  is the complex base band voltage,  $v_i$  s are the amplitudes of multipath waves and the  $\phi_i$  s are their corresponding phases. N is the chosen specular power component and  $v_{diff}$  is the diffused voltage.

The general form of envelope PDF of equation (1) for fading is given by,

$$f_R(r) = r \int_0^\infty J_0(vr) \exp\left(\frac{-v^2\sigma^2}{2}\right) \left[ \prod_{i=1}^N J_0(V_i v) \right] v dv \dots(2)$$

Where,  $J_0(V_i v)$  is a single characteristic function of the single specular wave with magnitude,  $V_i$  and

$$\text{Where, } D(x;K,\infty) = \frac{1}{2} \exp(\infty K) I_0\left(x\sqrt{2K(1-\infty)}\right) + \frac{1}{2} \exp(-\infty K) I_0\left(x\sqrt{2K(1+\infty)}\right)$$

$$K = \frac{\text{Specular power}}{\text{Diffused power}} = \frac{V_1^2 + V_2^2}{2\sigma^2} \text{ and}$$

$$\Delta = [\text{Convenient parameter}] = \frac{\text{Peak Specular Power}}{\text{Average Specular Power}} - 1 = \frac{2V_1V_2}{V_1^2 + V_2^2}$$

$$L \text{ is the order } \geq \frac{1}{2} K\Delta$$

This PDF contains the Rayleigh and Rician PDF's as special cases. When  $K=0$  its shows Rayleigh fading behavior and when  $\Delta = 0$  shows Rician fading behavior.

**SYSTEM MODEL**

The channel is considered as single user Gaussian channel with  $n_t$  no of transmitting antennas and  $n_r$  no of receiving antennas. So the channel is referred as  $n_t \times n_r$  MIMO channel. The channel can be modeled as [12],

$$y(i) = H(i)x(i) + n(i) \dots\dots\dots (4)$$

Where,  $y(i)$  is the complex  $n_r \times 1$  vector of receive signals at the receiving side,  $x(i)$  is the  $n_t \times 1$  vector of transmitting signals at the transmitting side at symbol time  $i$ . The matrix,  $H(i)$  represents a  $n_r \times n_t$  channel matrix with entries  $h(i)_{ij}$ , which represent the fading coefficient values at time  $i$  between  $n_t$  no. of transmitters and  $n_r$  no. of receivers.

The H can be written as[15],

$$H = H_R + jH_I \dots\dots\dots (5)$$

Where,  $H_R$  and  $H_I$  are independent  $n_r \times n_t$  real matrices with i.i.d complex Gaussian entries and equal variances. The distribution of the magnitude of the elements of H is assume to be Two Wave Diffused Power (TWDP) distribution which is given by the equation (2).

$\exp\left(\frac{-v^2\sigma^2}{2}\right)$  is the characteristics function of the diffused component with mean-squared voltage,  $2\sigma^2$ .

The approximate received signal envelope PDF over TWDP fading channel is given by the following equation under the condition  $N=2$  [two specular components] and  $\sigma$  [nonzero diffused components]:-

$$f_R(r) = \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2} - K\right) \sum_{i=1}^L a_i D\left(\frac{r}{\sigma}; K, \Delta \cos \frac{\pi(i-1)}{2L-1}\right) \dots\dots\dots(3)$$

The components of  $n(i)$  are independent, zero mean, circularly symmetric complex Gaussian noise with independent real and imaginary parts having equal variance at a time  $i$ .

**MIMO Channel Capacity**

The capacity can be defined as the maximum of the mutual information. If the channel is also known to the transmitting side then the channel capacity is given by [8],

$$C = E_H \left\{ \log_2 \det \left( I_R + \rho_i H H^\dagger \right) \right\} \dots\dots\dots (6)$$

Where  $\dagger$  indicates Hermitian Conjugate and  $I_R$  is an  $n_r \times n_r$  identity matrix and  $\rho_i$  is the power assigned to the  $i^{th}$  transmitter according to the water-filling algorithm and  $\mu$  is chosen to satisfy,

$$\rho_i = \left( \mu - \lambda_i^{-1} \right)^+ \dots\dots\dots(7)$$

“+” means  $\rho$  value always should be positive.

In the equation  $E_\lambda \{ . \}$  denotes the expectation over H. After doing singular value decomposition (SVD) equation (6) can be decomposed as,

$$C = E_{\lambda} \left\{ \sum_{i=1}^k \log_2 (1 + \rho_i \lambda_i) \right\} \quad \dots\dots\dots (8)$$

Where,  $E_{\lambda} \{ \cdot \}$  denotes the expectation over  $\lambda$ ;  $k$  is the rank of  $H$  ( $k \leq m$ );  $m = \min(n_t, n_r)$  and  $\lambda_i$  ( $i=1,2,\dots,k$ ) denotes the positive eigenvalues of  $W$ .

We may express 'W' as,

$$W = \begin{cases} HH^{\dagger}, n_r \leq n_t \\ H^{\dagger}H, n_r > n_t \end{cases} \quad \dots\dots\dots (9)$$

**OPRA technique and capacity analysis**

Optimum Power and Rate Adaptation(OPRA) technique is used by the traditional SISO and SIMO system for performance enhancement controlling the power and rate of data transmission at the transmitting side, depends on the knowledge of the channel which is achieved by the transmitter through a feedback path from the receiver. The channel capacity for this technique is given by [16],

$$C_{OPRA} = B \int_{\gamma_0}^{\infty} \log_2 \left( \frac{\gamma}{\gamma_0} \right) f_{\gamma}(\gamma) d(\gamma) \quad \dots\dots\dots (10)$$

$$C_{OPRA} = B \log_2 e \frac{e^{-K}}{2} \sum_{i=1}^L a_i \left[ \sum_{p1=0}^{\infty} \frac{(K(1-\alpha_i))^{p1}}{p1!} \sum_{t1=0}^{p1} \frac{\Gamma \left[ (t1), \left( \frac{K+1}{\gamma} \right) \gamma_0 \right]}{t1!} e^{-\alpha_i K} \right. \\ \left. + \sum_{p2=0}^{\infty} \frac{(K(1+\alpha_i))^{p2}}{p2!} \sum_{t2=0}^{p2} \frac{\Gamma \left[ (t2), \left( \frac{K+1}{\gamma} \right) \gamma_0 \right]}{t2!} e^{-\alpha_i K} \right] \quad (13)$$

Where,  $\Gamma(a, x)$  is an incomplete Gamma function and  $a$  indicate the coefficient of the order  $L$ . The values of different coefficients of different orders for TWDP fading

$$\frac{(K+1)}{2\gamma} e^{(-K)} \sum_{i=1}^L a_i \left[ \left[ e^{-\alpha_i K} \sum_{p1=0}^{\infty} \frac{1}{(p1!)^2} (K(1-\alpha_i))^{p1} \frac{1}{(K+1)\gamma_0} \Gamma \left[ (p1+1), \frac{(K+1)\gamma_0}{\gamma} \right] \right. \right. \\ \left. \left. + e^{-\alpha_i K} \sum_{p2=0}^{\infty} \frac{1}{(p2!)^2} (K(1+\alpha_i))^{p2} \frac{1}{(K+1)\gamma_0} \Gamma \left[ (p2+1), \frac{(K+1)\gamma_0}{\gamma} \right] \right] \right. \\ \left. - \left( e^{-\alpha_i K} \sum_{p1=0}^{\infty} \frac{1}{(p1!)^2} (K(1-\alpha_i))^{p1} \Gamma \left[ (p1), \frac{(K+1)\gamma_0}{\gamma} \right] + \right. \right. \\ \left. \left. + e^{-\alpha_i K} \sum_{p2=0}^{\infty} \frac{1}{(p2!)^2} (K(1+\alpha_i))^{p2} \Gamma \left[ (p2), \frac{(K+1)\gamma_0}{\gamma} \right] \right) \right] - 1 = 0 \quad \dots\dots\dots (14)$$

Where  $B$  is the channel bandwidth,  $f_{\gamma}(\gamma)$  is the PDF of the output SNR and  $\gamma_0$  is the optimal cutoff SNR, below which no transmission is allowed, has to satisfy the condition,

$$\int_{\gamma_0}^{\infty} \left( \frac{1}{\gamma_0} - \frac{1}{\gamma} \right) f_{\gamma}(\gamma) d(\gamma) = 1 \quad \dots\dots\dots (11)$$

For TWDP fading channel the SNR PDF can be calculated from the equation (3) as,

$$f_{\gamma}(\gamma) = \frac{(K+1)}{2\gamma} e^{(-K)} \sum_{i=1}^L a_i \left[ I_0 \left( \sqrt{\frac{(4K(K+1)(1-\alpha_i)\gamma)}{\gamma}} \right) e^{-\alpha_i K} e^{-\frac{(K+1)\gamma}{\gamma}} \right. \\ \left. + I_0 \left( \sqrt{\frac{(4K(K+1)(1+\alpha_i)\gamma)}{\gamma}} \right) e^{-\alpha_i K} e^{-\frac{(K+1)\gamma}{\gamma}} \right] \quad \dots\dots\dots (12)$$

Where,  $\gamma$  is the average SNR,  $\alpha_i = \Delta \cos \frac{\pi(i-1)}{2L-1} = \frac{2V_1V_2}{V_1^2 + V_2^2} \cos \frac{\pi(i-1)}{2L-1}$  and  $I_0(\cdot)$  is a Bessel function.

Putting equation (12) in (10) we find that the OPRA Channel capacity will be,

Where, taking  $\frac{(K+1)\gamma_0}{\bar{\gamma}} = x$  and following an approach given in [16], it has been obtained that  $x_0$  always lies in the interval  $[0, K+1]$ .

**Results and Discussion**

The obtained expressions for capacity over TWDP fading channel over optimum power and rate adaptation technique have been numerically evaluated for different parameters of interest and plotted for the purpose of illustration. Capacity (per unit bandwidth) vs. average SNR ( $\bar{\gamma}$ ) (in dB) of OPRA a scheme have been plotted in Fig. 1. For OPRA scheme and the value of  $g_0$  have been derived numerically from (14). Capacity analysis is done for a MIMO system over TWDP fading channel and results are obtain using rejection method of simulation. In

all the case discussed here, it is assumed that the CSI is available for both transmitter and receiver. Effect of different parameters of TWDP distribution for a MIMO system is studied in fig :2. From the plots it can be seen that the capacity is increase with the increase of SNR value as well as decrease of  $\Delta$  parameter in both cases. But comparing the cases we can clearly say that in case of OPRA when  $K=5$  and  $\Delta=0.5$  for SNR=10 dB the capacity is 3 bits/sec/Hz.

But for a  $2 \times 2$  MIMO system when  $K=5$  even for  $\Delta=0.8$  the capacity is 8 bits/sec/Hz. In this case for  $\Delta=0.5$  the value will further increase as the decrease of  $\Delta$  value leads to the increase of capacity in a TWDP fading environment. On the other hand with increase of antenna number the capacity increase continuously in case of a MIMO system.

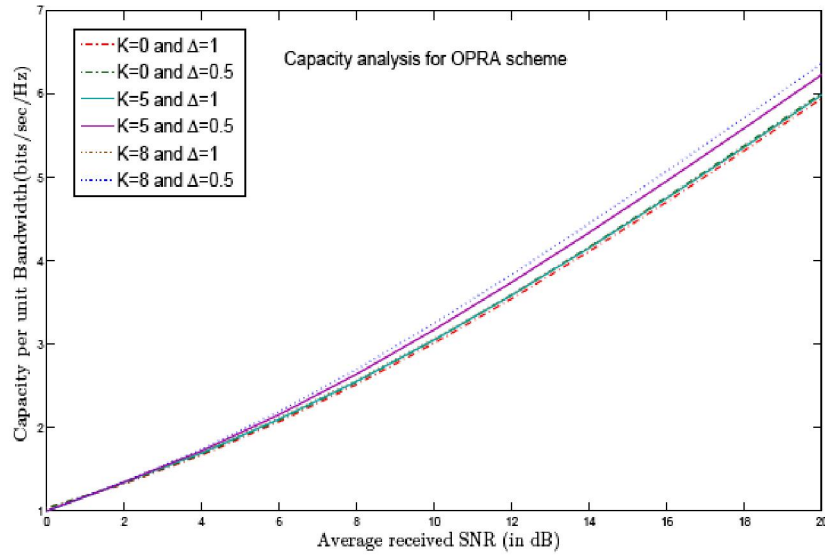


Fig. 1: Capacity for OPRA scheme over TWDP fading

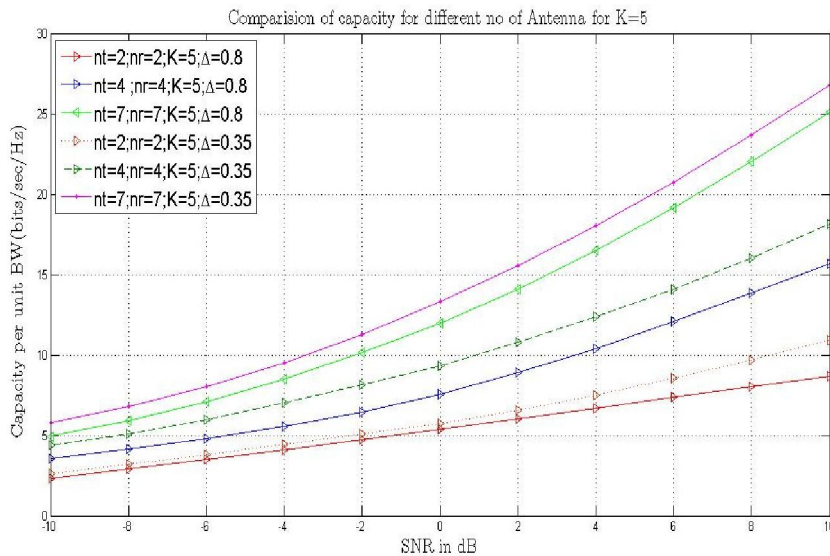


Fig 2: Capacity analysis for a MIMO system when K=5

**CONCLUSION**

In this paper, we analyze the capacity of a communication system over slow varying TWDP fading channels, for Optimum power and rate adaptation transmission technique and MIMO system. Numerically evaluated and simulated results have been plotted for different parameter of interest and compared with the available special case results. From this study it can be conclude that it is better to design a multiple antenna system than the implementation of adaptive policies in case of wireless system design from the performance point of view.

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