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ON KU-SEMIGROUPS

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ABSTRACT
The algebraic structure of semigroup with KU-algebra is called KU-semigroup and then we investigate some basic properties of this structure. We define the KU-semigroup and several examples are presented. Also, we study some types of ideals in this concept such as *S*-ideal, *k*- ideal and *P*-ideal. The relations between these types of ideals are discussed and few results for product *S*-ideals of product KU-semigroups are given. Furthermore, few results of some ideals in KU-semigroup under homomorphism are discussed.

KEY WORDS: KU-algebra; KU-semigroup; *S*-ideal; *k*- ideal; *P*-ideal.

INTRODUCTION
The notion of BCK and BCI- algebras are first introduced by Imai and Is´eki in 1966[2,3]. BCI-algebra is a generalization of BCK-algebra. In 1993, Jun and Hong [5] introduced a new class of algebras related to BCI-algebras and semigroups, called a BCI- semigroup. For the convenience of study, Jun *et al*.[6,4] renamed the BCI-semigroup as the IS-algebra and studied further properties of these algebras. Several authors studied the algebraic structures with semigroups see[1,7,8,12]. Prabpayak and Leerawat [13, 14] introduced a new algebraic structure which is called a KU-algebra. They gave the concept of homo morphisms of KU-algebras and investigated some related properties. In this paper, by combining KU-algebras and semi groups, we introduce the concept of KU-semigroups. We define some types of ideals in this concept such as *S*-ideal, *k*- ideal and *P*-ideal. The relations between these types of ideals are discussed and few results for product *S*-ideals of product KU-semigroups are given. Furthermore, few results of some ideals in KU-semigroup under homomorphism are discussed.

2. Preliminaries
In this section, we present some definitions and background about KU-algebra.

Definition 2.1.[13] Algebra $(X,*,0)$ is called a KU-algebra if it satisfies the following axioms:

(ku_1) $(x*y)*[(y*z))*(x*z)]=0$,

(ku_2) $x*0=0$,

(ku_3) $0*x=x$,

(ku_4) $x*y=0$ and $y*x=0$ implies $x Z y$ and

(ku_5) $x*x=0$.

On a KU-algebra X, we can define a binary relation $\leq$ by putting $x\leq y \Leftrightarrow y*x=0$. Then $(X,á)$ is a partially ordered set and 0 is its smallest element. Thus $(X,*,0)$ satisfies the following conditions. For all $x,y,z\in X$, we that

$(ku_{1'})$ $(y*z)*(x*z)\leq(x*y)$,

$(ku_{2'})$ $0\leq x$,

$(ku_{3'})$ $x\leq y, y\leq x$ implies $x=y$ and

$(ku_{4'})$ $y*x\leq x$.

Theorem 2.2 [9]. In a KU-algebra X. The following axioms are satisfied. For all $x,y,z\in X$,

(1) $x\leq y$ Imply $y*z\leq x*z$,

(2) $x*(y*z)=y*(x*z)$,for all $x,y,z\in X$,

(3) $((y*x)*x)\leq y$.

Definition 2.3 [14]. A non-empty subset S of a KU-algebra $(X,*,0)$ is called KU-sub algebra of X if $x*y\in S$ whenever $x,y\in S$.

Definition 2.4 [13]. A non-empty subset I of a KU-algebra $(X,*,0)$ is called an ideal of X if for any $x,y\in X$, then

(i) $0\in I$ and

(ii) $x*y, x\in I$ imply that $y Č I$.

Definition 2.5 [13]. Let I be a non empty subset of a KU-algebra X. Then I is said to be a KU-ideal of X, if (I_1) $0\in I$ and (I_2) $\forall x,y,z\in X$, $x*(y*z)\in I$ and $y\in I$ imply that $x*z\in I$.

Definition 2.6 [11]. A KU-algebra $(X,*,0)$ is said to be a KU-commutative if it satisfies: for all x,y in X, $(y*x)*x=(x*y)*y$, where $x\wedge y=(y*x)*x$, i.e. $x\wedge y=y\wedge x$.

Example 2.7 [11]. Let $X=\{0,a,b,c,d,e\}$ be a set, with the operation $*$ defined by the following table:

$*$	0	a	b	c	d	e
0	0	a	b	c	d	e
a	0	0	b	c	b	c
b	0	a	0	b	a	d
c	0	a	0	0	a	a
d	0	0	0	b	0	b
e	0	0	0	0	0	0

Then $(X,*,0)$ is a KU-algebra and KU-commutative.

3. A KU-algebra with semigroups

Now, we give the definition and properties of a KU-algebra with semigroups.

Definition3.1. A KU-semigroup is a nonempty set X with two binary operations $*$, $\circ$ and constant 0 satisfying the following axioms

(I) $(X,*,0)$ is a KU-algebra,

(II) $(X,\circ)$ is a semigroup,

(III) The operation $\circ$ is distributive (on both sides) over the operation $*$, i.e.
$x\circ(y*z)=(x\circ y)*(x\circ z)$ *and* $(x*y)\circ z=(x\circ z)*(y\circ z)$, for all $x,y,z\in X$.

Example 3.2. Let $X=\{0,1,2,3\}$ be a set. Define $*$ - operation and $\circ$ - operation by the following tables

$*$	0	1	2	3
0	0	1	2	3
1	0	0	0	2
2	0	2	0	1
3	0	0	0	0

$\circ$	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

By routine calculations, it is easy to show that $(X,*,\circ,0)$ is a KU-semigroup

Proposition 3.3. Let $(X,*,\circ,0)$ be a KU-semigroup . The following axioms are satisfied. For all $x,y,z\in X$,

(1) $x\circ 0=0$ and $0\circ x=0$,

(2) If $x\le y$ imply $z\circ x\le z\circ y$ and $x\circ z\le y\circ z$,

(3) $x\circ(y\wedge z)=(x\circ y)\wedge(x\circ z)$ *and* $(x\wedge y)\circ z=(x\circ z)\wedge(y\circ z)$.

Proof. (1) $x\circ 0=x\circ(0*0)=(x\circ 0)*(x\circ 0)=0$ and $0\circ x=(0*0)\circ x=(0\circ x)*(0\circ x)=0$.

(2) Let $x\le y$, then we have $y*x=0$. Now, $(z\circ y)*(z\circ x)=z\circ(y*x)=z\circ 0=0$, it follows that $z\circ x\le z\circ y$. Similarly, we can prove that $x\circ z\le y\circ z$.

(3) we have

$x\circ(y\wedge z)=x\circ[(z*y)*y]=[x\circ(z*y)]*(x\circ y)=[(x\circ z)*(x\circ y)]*(x\circ y)$

$=(x\circ y)\wedge(x\circ z)$

By the same way, we can prove that $(x\wedge y)\circ z=(x\circ z)\wedge(y\circ z)$.

Proposition 3.4. Let $(X,*,\circ,0)$ be a KU-semigroup and $x,y,z\in X$. Then the following properties hold

(i) $x\circ(0*y)=(0*x)\circ y=x\circ y$,

(ii) If $x\circ y=0$, then $x\circ(y*z)=x\circ z$,

(iii) If $x\circ z=0$, then $(x*y)\circ z=y\circ z$,

(iv) $x\circ[y*(0*z)]=(x\circ y)*[0*(x\circ z)]$ and $[y*(0*z)]\circ x=(y\circ x)*[0*(z\circ x)]$.

Proof. Clear.

In the following, we define a new concept which is called a *strong* KU-semigroup.

Definition 3.5. Let $(X,*,\circ,0)$ be a KU-semigroup. If $x*y=(y\circ x)*y$, for all $x,y\in X$, then $(X,*,\circ,0)$ is called a *strong* KU-semigroup.

Definition 3.6. an element e is called a *unity* in a KU-semigroup if $e\circ x=x\circ e=x$, for all $x\in X$. If X is a *strong* KU-semigroup with a unity e, then e is the greatest element in X, since $e*x=(x\circ e)*x=x*x=0$, for all $x\in X$.

Theorem 3.7. Let $(X,*,\circ,0)$ be a *strong* KU-semigroup. Then the following properties hold, for all $x,y\in X$,

(1) $x\circ y\le y$,

(2) $x\le y \Leftrightarrow x\le x\circ y$,

(3) $y*(x\circ y)=(y*x)\circ y$.

Proof. (1) Let $x,y\in X$

$y*(x\circ y)=[(x\circ y)\circ y]*(x\circ y)=[x\circ(y\circ y)]*(x\circ y)=x\circ[(y\circ y)*y]$

$=x\circ(y*y)=x\circ 0=0.$

Hence $x\circ y\le y$.

(2) if $x\le y$, then $y*x=0$. It follows that $(x\circ y)*x=0$, hence $x\le x\circ y$, conversely it is clear.

(3) Let $x,y\in X$, we have

$y*(x\circ y)=[(x\circ y)\circ y]*(x\circ y)=[(x\circ y)*x)]\circ y=(y*x)\circ y$.

Theorem3.8. Let $(X,*,\circ,0)$ be a *strong* KU-semigroup and $(x*y)*y=(y*x)*x$. Then $x\wedge y=x\circ y$.

Proof. Let $x,y\in X$, we have

$x\wedge y=(y*x)*x=[(x\circ y)*x]*x$

$=[x*(x\circ y)]*(x\circ y)=x\circ[e*y]*(x\circ y)$

$=(x\circ 0)*(x\circ y)=0*(x\circ y)=(x\circ y).$

4. Some types of ideals in a KU-*semigroup*

In this part , we introduce some types of ideals in a KU-*semigroup* and we prove that some intresting properties.

Definition 4.1. A nonempty subset A of X is called a *sub* KU-*semigroup* of X, if $x*y, x\circ y\in A$, for all $x,y\in A$.

Definition 4.2. A non empty subset A of a KU-semigroup X is called an *S-ideal* of X if

i) A is an ideal of X,

ii) For all $x\in X$, $a\in A$, we have $x\circ a\in A$ and $a\circ x\in A$.

Example 4.3. Let $X=\{0,a,b,c\}$ be a set. Define $*$ - operation and $\circ$ - operation by the following tables

*	0	a	b	c
0	0	a	b	c
a	0	0	0	c
b	0	a	0	c
c	0	0	0	0

∘	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Then $(X,*,\circ,0)$ is a KU-semigroup and $\{0,b\}$ is an *S*-ideal of a KU-semigroup.

Theorem 4.4. In a KU-semigroup X . Any *S-ideal* of X is a *sub* KU-*semigroup* of X and the converse is not true.

Proof. Let A be an *S*-ideal of X. Then $0\in A$ and $y*(x*y)=0$, for all x,y Č X. Thus for $x,y\in A$ we have $y*(x*y)\in A$, which implies $x*y\in A$. And by Theorem 3.7 (1) we have, for all $x,y\in X$ $x\circ y\le y\Rightarrow y*(x\circ y)=0$. It follows that for $x,y\in A$ and $0=y*(x\circ y)\in A$ implies that $x\circ y\in A$. Hence A is a *sub* KU-*semigroup*.

The converse of this Theorem, it is easy to show that in example 4.3 $B=\{0,a\}$ is a *sub* KU-*semigroup* but not *S-ideal, since* $a*b=0\in B$ and $a\in B$ but $b\notin B$.

Definition 4.5. A non empty subset A of a KU-semigroup X is called an *k-ideal* of X if

i) A is an KU-ideal of X ,

ii) For all $x\in X$, $a\in A$, we have $x\circ a\in A$ and $a\circ x\in A$.

Theorem 4.6. Let X be a KU-semigroup. A non empty subset A of a KU-semigroup X is an *k-ideal* of X if and only if A is an *S*-ideal of X .

Proof. Let A be an *k-ideal* in *X*; it is clear that $0\in A$. Since for any $x,y,z\in X$, $(x*(y*z))\in A, y\in A \Rightarrow (x*z)\in A$, then by put $x=0$, we obtain $(y*z)\in A, y\in A\Rightarrow z\in A$. Hence I is an *S*-ideal of X .

Conversely, let A *be an S- ideal* of X , then $0\in A$. Now, if $(x*(y*z))\in A, y\in A$, then (by Th.2.2 (2)) $(y*(x*z))\in A$ and $y\in A$, since A *is an S-ideal* of X , thus $(x*z)\in A$, therefore A *is an k-ideal* of X .

Combining Theorem 4.4 and Theorem 4.6, we have the following corollary.

Corollary 4.7. Any *k* -ideal of a KU-semigroup X is a *sub* KU-*semigroup.*

Definition 4.8. A non empty subset A of a KU-semigroup X is called an *P-ideal* of X if

(p_1) For any $x,y,z\in X$, $z*(x*y)\in A$ and $z*x\in A\rightarrow z*y\in A$.

(p_2) For all $x\in X$, $a\in A$, we have $x\circ a\in A$ and $a\circ x\in A$.

Example 4.9. Let $X=\{0,a,b,c\}$ be a set. Define $*$ -operation and $\circ$ -operation by the following tables

*	0	a	b	c
0	0	a	b	c
a	0	0	a	c
b	0	0	0	c
c	0	a	b	0

∘	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	b	c
c	0	0	c	b

Then $(X,*,\circ,0)$ is a KU-semigroup and $\{0,a,b\},\{0,c\}$ are *P*-ideals of a KU-semigroup.

Lemma 4.10. In a KU-semigroup X . Any *P-ideal* is an *S-ideal* .

Proof. Let A be an *P-ideal* of X . we put $z=0$ in Definition4.8. (p_1) to obtine the result.

Propositon 4.11. Let A_1 and A_2 be two *S*-ideals of a KU-semigroup X , then $A_1\cap A_2$ is an *S*-ideal of X .

Proof . Let A_1 and A_2 be two *S*-ideals of X, for any $x\in X$ and $a\in A_1\cap A_2\Rightarrow a\in A_1, a\in A_2$. Then $x\circ a, a\circ x\in A_1$ and $x\circ a, a\circ x\in A_2$ implies that $x\circ a, a\circ x\in A_1\cap A_2$

Now, let $x*y\in A_1\cap A_2$ and $x\in A_1\cap A_2$, then $x*y\in A_1, x\in A_1$ and $x*y\in A_2, x\in A_2$ and since A_1, A_2 are *S*-ideals of X implies that $y\in A_1$ and $y\in A_2$. Hence $y\in A_1\cap A_2$, it follows that $A_1\cap A_2$ is an *S*-ideal of X .

Proposition 4.12. Let A_1 and A_2 be two *S*-ideals of a KU-semigroup , then $A_1\cup A_2$ is an *S*-ideal of X if $A_1\subseteq A_2$ or $A_2\subseteq A_1$.

Proof. Suppose A_1 and A_2 be two *S*-ideals of a KU-semigroup *X* that without loss of generality we may assume that $A_1\subseteq A_2$, then $A_1\cup A_2=A_2$

Since A_2 is an *S*-ideal, So $A_1\cup A_2$ is an *S*-ideal.

Similarly, if $A_2\subseteq A_1$, then $A_1\cup A_2=A_1$ and Since A_1 is an *S*-ideal, So $A_1\cup A_2$ is an *S*-ideal.

Now, the product of two ideals in a KU-semigroup is given in the following definition

Definition 4.13. Let A_1 and A_2 be two an *S*-ideals in a KU-semigroup X . The product $A_1\times A_2=\{(a,b): a\in A_1, b\in A_2\}$ and the binary operations " $\circ$ " and " $*$ " on $A_1\times A_2$ are define by the following : for all (a_1,b_1) , $(a_2,b_2)\in A_1\times A_2$.

$(a_1,b_1)\circ(a_2,b_2)=(a_1\circ a_2, b_1\circ b_2)$,

$(a_1,b_1)*(a_2,b_2)=(a_1*a_2, b_1*b_2)$.

Proposition 4.14. Let A_1 and A_2 be two *S*-ideals of KU-semigroup *X*. Then $A_1\times A_2$ is an *S*-ideal of $X\times X$.

Proof . Let $(x_1,x_2)\in X\times X$ *and* $(a_1,a_2)\in A_1\times A_2$.

Then $(x_1,x_2)\circ(a_1,a_2)=(x_1\circ a_1, x_2\circ a_2)$. Since $x_1\circ a_1\in A_1$ and $x_2\circ a_2\in A_2$

Then $(x_1\circ a_1, x_2\circ a_2)\in A_1\times A_2$. So $(x_1,x_2)\circ(a_1,a_2)\in A_1\times A_2$ and by the same way, we prove that $(a_1,a_2)\circ(x_1,x_2)\in A_1\times A_2$

Now,

Let $(x * y) \in A_1 \times A_2$ and $x \in A_1 \times A_2$, where $x = (x_1, x_2)$, $y = (y_1, y_2) \in X \times X$

If $[(x_1, x_2) * (y_1, y_2)] \in A_1 \times A_2$ and $(x_1, x_2) \in A_1 \times A_2$.

Then $(x_1 * y_1, x_2 * y_2) \in A_1 \times A_2$ and $(x_1, x_2) \in A_1 \times A_2$, it follows that

$x_1 * y_1 \in A_1$, $x_2 * y_2 \in A_2$ and $x_1 \in A_1$, $x_2 \in A_2$.

Then $y_1 \in A_1$ and $y_2 \in A_2$ [Since A_1, A_2 are two S-ideals], So $(y_1, y_2) \in A_1 \times A_2$,

then $\in A_1 \times A_2$. Hence $A_1 \times A_2$ is an S-ideal.

5- homomorphism of KU-semigroup

Let X and X' be two KU-semigroups. A mapping $\phi : X \to X'$ is called a KU-semigroup homomorphism if $\phi(x * y) = \phi(x) * \phi(y)$ and $\phi(x \circ y) = \phi(x) \circ \phi(y)$, for all $x, y \in X$. The set $\{x \in X \mid \phi(x) = 0\}$ is called the kernel of ϕ and denote by $ker\phi$. Moreover, the set $\{\phi(x) \in X' \mid x \in X\}$ is called the image of ϕ and denote by $im\ \phi$.

Proposition 5.1. Let $\phi: X \to X'$ be a KU-semigroup homomorphism. Then for all $x, y \in X$.

i) $\phi(0) = 0'$.

ii) $x \le y$ imply $\phi(x) \le \phi(y)$.

iii) $\phi(x \wedge y) = \phi(x) \wedge \phi(y)$.

Proof: (i) Suppose that x is an element of X. Then

$\phi(0) = \phi(x * x) = \phi(x) * \phi(x) = 0$

(ii) Let $x \le y$. Then we have $y * x = 0$. Thus we have

$0 = \phi(0) = \phi(y * x) = \phi(y) * \phi(x)$, and so $\phi(x) \le \phi(y)$.

(iii)$\phi(x \wedge y) = \phi((y * x) * x) = \phi(y * x) * \phi(x) = [\phi(y) * \phi(x)] * \phi(x) = \phi(x) \wedge \phi(y)$.

Proposition 5.2. Let $\phi: X \to X'$ be a KU-semigroup homomorphism and $^{-1}(0) = \{0\}$. Then if $\phi(x) \le \phi(y)$ imply $x \le y$.

Proof: If $\phi(x) \le \phi(y)$, then we have $\phi(y) * \phi(x) = \phi(y * x) = 0$ and so $y * x$ is an element of $\ddot{\psi}^{-1}(0)$. Hence $y * x = 0$, it follows that $x \le y$.

Proposition 5.3. Let $\phi: X \to X'$ be a KU-semigroup homomorphism. Then $\ker\phi$ is an S-ideal of X.

Proof . Let $\phi: X \to X'$ be a KU-semigroup homomorphism and $x, y \in X$ such that $x * y \in \ker(\phi)$ and $x \in ker\phi$, so $\phi(x * y) = 0$ and $\phi(x) = 0$, it follows that

$\phi(x) * \phi(y) = 0$ and $\phi(x) = 0$, then $0 * \phi(y) = 0$ imply $\phi(y) = 0$ and

$y \in ker\phi$. Hence $ker\phi$ is an ideal of KU-algebra.

Now, let $x \in X$ and $a \in \ker\phi$ so $\phi(a) = 0$, then

$\phi(x \circ a) = \phi(x) \circ \phi(a) = \phi(x) \circ 0 = 0$. Hence $x \circ a \in ker\phi$ and

$\phi(a \circ x) = \phi(a) \circ \phi(x) = 0 \circ \phi(x) = 0$. Hence $a \circ x \in ker\phi$. So $\ker\phi$ is an S-ideal of KU-semigroup.

Proposition 5.4. Let $\phi: X \to X'$ be a KU-semigroup epimorphism and A be an S-ideal in X. Then $\phi(A)$ is an S-ideal in X'.

Proof . Let A be an S-ideal of , and suppose that $\phi(x), \phi(y) \in \phi(A)$ for some $x, y \in A$ such that $\phi(x) * \phi(y) \in \phi(A)$ and $\phi(x) \in \phi(A)$ to prove $\phi(y) \in \phi(A)$.

Since ϕ is a homomorphism, then $\phi(x) * \phi(y) = \phi(x * y)$ and since $\phi(x) \in \phi(A)$

Thus $x * y \in A, x \in A \to y \in A$ [since A is an ideal]

Therefore $\phi(y) \in \phi(A)$. Hence $\phi(A)$ is an ideal in X'.

Now, $\phi(a) \in \phi(A)$ and $y \in X'$ where $a \in A$.

Since ϕ onto, then there exists $x \in X$ such that $\phi(x) = y$. To prove $y \circ \phi(a) \in \phi(A)$

Since $\forall x \in X$ and $\in A$, we have $x \circ a \in A$ and $a \circ x \in A$ [A is an S-ideal]. So $\phi(x \circ a) \in \phi(A)$, and $\phi(a \circ x) \in \phi(A)$. Hence $\phi(x \circ a) = \phi(x) \circ \phi(a) = y \circ \phi(a) \in \phi(A)$ and $\phi(a \circ x) = \phi(a) \circ \phi(x) = \phi(a) \circ y \in \phi(A)$. Then $\phi(A)$ is an S-ideal in X'.

Theorem 5.5. Let $\phi: X \to X'$ be a KU-semigroup homomorphism. If A is an S-ideal of X', then $\phi^{-1}(A) = \{x \in X \mid \phi(x) \in A\}$ is an S-ideal of X containing $ker\phi$.

Proof- Let $\phi: X \to X'$ be a KU-semigroup homomorphism and $x, y \in X$ such that

$x * y \in \phi^{-1}(A)$ and $x \in \phi^{-1}(A)$, then $\phi(x) * \phi(y) = \phi(x * y) \in A$ and $\phi(x) \in A$. and since A is an S-ideal, it follows that $\phi(y) \in A$. That is $y \in \phi^{-1}(A)$, therefore $\phi^{-1}(A)$ is an ideal of X. Moreover $\{0\} \subseteq A$ implise that $ker\phi = \phi^{-1}(\{0\}) \subseteq \phi^{-1}(A)$.

Now, Suppose that A is an S-ideal of X'. Let $x \in X$ and $y \in \phi^{-1}(A)$, then

$\phi(y) \in A$ and $\phi(x \circ y) = \phi(x) \circ \phi(y) \in A$ and $\phi(y \circ x) = \phi(y) \circ \phi(x) \in A$. Hence $x \circ y, y \circ x \in \phi^{-1}(A)$ and then $\phi^{-1}(A)$ is an S-ideal of X containing $ker\phi$.

Proposition 5.6. Let $\phi: X \to X'$ be a KU-semigroup epimorphism and A be an P-ideal in X. Then $\phi(A)$ is an P-ideal in X'.

Proof. Let A be an P-ideal of .

Now, Suppose that $\phi(x), \phi(y), \phi(z) \in \phi(A)$ for some $x, y, z \in A$ such that

$\phi(z) * [\phi(x) * \phi(y)] \in \phi(A)$ and $(z) * \phi(x) \in \phi(A)$.

Since ϕ is a homomorphism, then $\phi[z * (x * y)] \in \phi(A)$ and $\phi(z * x) \in \phi(A)$

Thus $* (x * y) \in A$, $z * x \in A \to z * y \in A$ [since A is P-ideal]

Therefore $\phi(z) * \phi(y) = \phi(z * y) \in \phi(A)$.

Now, let $\phi(a) \in \phi(A)$ and $y \in X'$, where $a \in A$.

Since ϕ onto, then there exists $x \in X$ such that $\phi(x) = y$. To prove $y \circ \phi(a) \in \phi(A)$
Since $\forall x \in X$ and $\in A$, we have $x \circ a \in A$ and $a \circ x \in A$ [A is an P-ideal]. So $\phi(x \circ a) \in \phi(A)$, and $\phi(a \circ x) \in \phi(A)$. Hence $\phi(x \circ a) = \phi(x) \circ \phi(a) = y \circ \phi(a) \in \phi(A)$ and $\phi(a \circ x) = \phi(a) \circ \phi(x) = \phi(a) \circ y \in \phi(A)$.
Hence $\phi(A)$ is an P-ideal in X'.

CONCLUSION

A new concept of KU-algebras which is called a KU-semigroup . This concept we obtained by a combining KU-algebras and semigroups and discussed few properties of this concept and investigate some related results. Moreover, some types of ideals in KU-semigroup are studied and the product of *S*-ideals to a KU-semigroup is established. The notion of a homomorphism of a KU-semigroup is discussed. The main purpose of our future work is to study of a fuzzy KU-semigroup and a generlazation of a fuzzy KU-semigroup such as a bipolar of a fuzzy KU-semigroup and interval value of a fuzzy KU-semigroup. Also, we can introduce the notion of graph for a KU-semigroup.

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